Integrating Belief Functions with Model-based Diagnosis for Fault Management

Xin Hong, Weiru Liu and William Scanlon*
School of Computing and Information Engineering
University of Ulster at Coleraine, BT52 1SA, UK
Phone: +44-28-70324860, Fax: +44-28-70324916
email:{x.hong,w.liu}@ulster.ac.uk
*School of Electrical and Electronic Engineering
Queen's University of Belfast, Belfast, BT9 5AH, UK
Phone: +44-28-90274519, Fax: +44-28-667023
email: w.scanlon@ee.qub.ac.uk

ABSTRACT: The simulation model of a device/system available from design implies the use of model-based approach in the manufacturing fault diagnosis. The model-based diagnosis uses knowledge of the structure and behaviour of a device and observations of the device's behaviour to discover the causes of malfunctions. The major difficulty of model-based diagnosis is to eliminate hypotheses accounting for all symptoms. Belief function theory of evidence provides a numerical mechanism to narrow the hypothesis set with the accumulation of evidence. This paper presents an approach to integrating belief functions with model-based diagnosis to improve the efficiency of the hypothesis elimination. When more than one component is diagnosed faulty, further measurements need to be carried out to eliminate less likely faulty components. Our belief function model of model based diagnosis uses beliefs committed to hypotheses to help select the best next measurement to perform in order to eliminate less suspected components.

KEYWORDS: Fault diagnostics, model-based diagnosis, belief functions, belief updating

1. INTRODUCTION

In recent years, the telecommunications industry has grown significantly along with increasing demands for higher product performance and reliability, greater levels of functionality and more competitive prices. In order for manufacturers to be able to deliver quality products in reasonable time, an efficient testing procedure has to be provided. A major function associated with manufacturing test procedure is fault diagnosis. Diagnosis can be generally described as determining the cause of differences between observed behaviour and expected behaviour of a system in complex situations. In terms of electronic manufacturing fault diagnosis, the objective is to find the faulty component(s) that can explain the malfunctioning of the system.

With computer-aided design techniques the design department avails the simulation model of a system to be diagnosed in the manufacturing floor. This features the use of model based approach in the manufacturing fault diagnostics. Model-based diagnosis [1], based on the fundamental knowledge of a domain (also called first principles), provides a mechanism of diagnosis to identify the malfunctioning components in a well-designed device (*i.e.* a 'system'), which prevent the device from functioning properly.

The model-based diagnosis generally processes in two main stages. The first stage aims to identify sets of components (called hypotheses) that can account for all symptoms. When more than one suspect component is identified, the second stage is then to eliminate the hypotheses by collecting additional evidence. Many techniques have been developed to carry out the tasks involved in these two stages. For example, in respect of creating hypotheses to explain the symptoms, an assumption-based truth maintenance system (ATMS) [2] has presented an effective and efficient mechanism to pick up the probable candidates by keeping track of the dependence among components, especially those components used in assumptions. It is usually the case that there is more than one suspect that accounts for all symptoms. So further measurements (or called tests) need to be performed to eliminate less possible suspects. However, this process is incremental and additional observations do not guarantee a shorter list of suspects. de Kleer [3] provided a probabilistic approach to select the best next measurement under the premise of equal prior probabilities for each component by using the minimum entropy technique.

Among the numerical reasoning methods, the belief function theory of evidence [4], [5] is efficient at guiding the inference even under uncertainty by using degrees of belief. In this paper, we integrate belief functions with model-based diagnosis to provide quantitative analyses of the best next measurement. An efficient process of belief calculation is also presented.

The paper is organised as follows. Section 2 briefly introduces the principle of model-based diagnosis. Section 3 outlines the belief function theory of evidence. Section 4 presents our approach to integrating belief functions with model-based diagnosis. Section 5 uses an example to illustrate how our proposed method eliminates less possible candidates. Finally, section 6 summarises the paper.

2. MODEL-BASED DIAGNOSIS

Model-based diagnosis refers to diagnosis from first principles [1], where one begins with descriptions of the structure and behaviours of the individual components comprising a device (in terms of model), together with observations of the device's behaviours. The diagnostic problem arises when observations of the device's actual performance, typically measurements at its inputs and outputs, conflict with the device's expected behaviours. The diagnostic task is then to determine which of the components could have failed in a way that accounts for all of the discrepancies observed.

The diagnostic procedure is guided by symptoms that are differences between predictions and observations [6]. Predictions can be achieved through the inference procedure upon constraints of functions of components and the structure of the device. Observations are obtained by measurements. Model-based diagnosis first starts with the assumptions that system components are working correctly. When a symptom occurs, one or more assumptions obviously are inconsistent. In Reiter's theory of model-based diagnosis, a candidate is defined as a set of components that might be malfunctioning. A candidate is also a particular hypothesis that can explain a symptom. The set of candidates consistent with the observation makes up a candidate space. The size of the candidate space grows exponentially with the number of components. Reiter's framework does not provide a mechanism to eliminate candidates.

3. BELIEF FUNCTION THEORY OF EVIDENCE

The belief function theory of evidence is a numerical reasoning method, first proposed by Dempster [4] and further developed by Shafer [5], which represents the evidence in the form of generalised probabilities. In the belief function theory of evidence, a problem is described in terms of an environment that consists of all possible situations (or values). The elements of the environment are mutually exclusive and exhaustive. The exhaustive set of mutually exclusive elements is referred to as a *frame of discernment* denoted as Θ. A mass function m: $2^{\Theta} \to [0,1]$, represents the distribution of a unit of belief over a frame, Θ, satisfying the following two conditions:

(1) $m(\Phi) = 0$, where Φ is an empty set.

(2) $\Sigma_{A\subseteq\Theta}\, m(A)=1$.

A *belief function* over Θ is a function Bel: $2^{\Theta} \to [0,1]$, satisfying $Bel(A) = \Sigma_{B\subseteq A}\, m(B)$. $Bel(A)$ is the total belief committed to A.

The belief function theory of evidence aggregates degrees of belief with new pieces of evidence. When a new piece of evidence T is observed, the belief function $Bel(\cdot)$ is updated to the conditioned belief function $Bel(\cdot/T)$. The conditioned belief function can be calculated by using prior belief functions as belows [7],

$$Bel(S/T) = \frac{Bel(S \cap \sim T) - Bel(\sim T)}{1 - Bel(\sim T)}\ . \qquad (1)$$

4. BELIEF FUNCTION MODEL OF MODEL-BASED DIAGNOSIS

4.1 TERMS AND NOTATIONS

We call a set, which collects all possible outcomes of a system, as a state space (also a frame of discernment) N, each element of N represents a state of the system. For a system with one component E, a situation of the component is a possible outcome of the system, so the state space N of the system is comprised of all possible situations of the component. For example, when $E = \{e, \sim e\}$, e represents that the component E is intact, $\sim e$ represents that the component E is faulty (not intact). For a system with n components, the state space N is the Cartesian product of n state spaces N_i for n subsystems, $N = N_1 \times \cdots \times N_n$, the size of the state space is $|N_1|\times\ldots\times|N_n|$. When a symptom is observed, all states in consistency with the symptom compose a diagnosis set N_d, and rest of states that are inconsistent with the symptom make up a conflict set N_c, $N=N_d \cup N_c$, $N_d \cap N_c=\Phi$. A hypothesis is about the system states that contribute the symptom [8]. All states implicated by the hypothesis set up a subset of the state space, called as the hypothesis set N_h.

4.2 IMPORTANT PROPERTIES

Now let us explore the potential properties of the state space N, the conflict set N_c, the diagnosis set N_d and the hypothesis set N_h. In the state space N_i of a system with a component E_i, each state corresponds to a subset of the state space N of the system E with n components including component E_i, where $\{e_i\}$ corresponds to

$$N_{\{e_i\}} = \{\{e_i\} \times N_j \times \cdots \times N_n\},\ j = 1, \cdots, n, i \neq j,$$

and $\{\sim e_i\}$ corresponds to

$$N_{\{\sim e_i\}} = \{\{\sim e_i\} \times N_j \times \cdots \times N_n\},\ j = 1, \cdots, n, i \neq j.$$

Theorem 1. $\{N_{\{e_i\}}, N_{\{\sim e_i\}}\}$ is a partition of N.

Proof: $N_{\{e_i\}} \subset N$, $N_{\{\sim e_i\}} \subset N$, $N= N_{\{e_i\}} \cup N_{\{\sim e_i\}}$, and $N_{\{e_i\}} \cap N_{\{\sim e_i\}} =\Phi$. So $\{N_{\{e_i\}}, N_{\{\sim e_i\}}\}$ is a partition of N.

Corollary 1. For a hypothesis h and the complement $\sim h$, if N_h and $N_{\sim h}$ represent the subsets of N related to h and $\sim h$ respectively, then

$\{N_h, N_{\sim h}\}$ is a partition of N.

Theorem 2. If we have m minimal conflicts c_i by observations, c_i corresponds to a conflict space N_{c_i}, the conflict space N_c is related to $c = \bigcup_{i=1}^{m} c_i$, then

$$N_c = \bigcup_{i=1}^{m} N_{c_i}.$$

Proof: If c_i is a minimal conflict, c_i corresponds to a set $\{x: x\in N_{c_i}\}$. The conjunction set $c = \bigcup_{i=1}^{m} c_i$ corresponds to a conjunction set $\cup\{x: x\in N_{c_i}\}$, $\cup\{x: x\in N_{c_i}\}$ equals to $\{x: x\in \cup N_{c_i}\}$, so $c = \bigcup_{i=1}^{m} c_i$ corresponds to $N_c = \bigcup_{i=1}^{m} N_{c_i}$.

Theorem 3. If $N=N_1\times\cdots\times N_n$, $h=\{h_1, \cdots, h_m\}$, $h_i \in N_i$, N_{h_i} is the set corresponded by h_i, then we have a set N_h corresponded by h,

$$\begin{aligned} N_h &= \{h_1 \times \cdots \times h_m \times N_{m+1} \times \cdots N_n\} \\ &= N_{h_1} \cap \cdots \cap N_{h_m}. \end{aligned}$$

Proof: A state $x=(x_1, \ldots, x_n)$, $x\in N_h$, then $x_1=h_1, \ldots, x_m=h_m$. Thus $x\in N_{h_i}$, …, and $x\in N_{h_m}$. Thus $x \in \bigcap_{i=1}^{m} N_{h_i}$.

For the convenience of discussion later on, two useful theorems from the pure set theory [9] are presented here,

Theorem 4. If a set $A=A_1\cup\cdots\cup A_n$, then $A=B_1\cup\cdots\cup B_n$, $B_i \cap B_j=\Phi$,

$$B_1=A_1,\ B_i=A_i \cap \sim A_1 \cap \cdots \cap \sim A_{j\neq i}.$$

Theorem 5. If α, $A_1, \ldots A_k$ are sets,

$$\alpha \cap \sim(A_1 \cap \cdots \cap A_k) = (\alpha \cap \sim A_1) \cup (\alpha \cap A_1 \cap \sim A_2) \cup \cdots \cup (\alpha \cap A_1 \cap A_2 \cap \cdots \cap \sim A_k).$$

4.3 BELIEF FUNCTION MODEL OF MODEL-BASED DIAGNOSIS

Suppose that a system has n independent components, each of which has only two states $\{e_i\}$(intact) and $\{\sim e_i\}$(faulty or not intact). Before any observation carried out, there is a prior probability distribution on E_i presented by $p_{\{e_i\}}$ and $p_{\{\sim e_i\}}$, and $p_{\{e_i\}}+p_{\{\sim e_i\}}=1$. A prior probability distribution on N is then presented by a probability of a state $x=\{x_1,\ldots,x_n\}$, where x_i is the state of ith component, $x_i = e_i$ when the component is intact, otherwise $x_i = \sim e_i$,

$$p(x)=\prod_{x_i=e_i} p_{\{e_i\}} \prod_{x_i=\sim e_i} p_{\{\sim e_i\}} = \prod_{x_i=e_i} p_{\{e_i\}} \prod_{x_i=\sim e_i} (1-p_{\{e_i\}}). \tag{2}$$

For a subset $A\subseteq N$,

$$P(A)=\sum_{x\in A} p(x).\ \ ([10]) \tag{3}$$

For a hypothesis h corresponding to a hypothesis set, the belief on the hypothesis is represented by the total beliefs over the hypothesis set $Bel(N_h)$. Obviously $P(N)=1$. The prior belief function on N as the generalised probability function can be calculated as:

$$Bel(N_h)=\sum_{A\subseteq N_h} m(A)=\sum_{x\in N_h} p(x)=P(N_h). \tag{4}$$

Theorem 6 [5]. A belief function on a partition $\{N_s, N_{\sim s}\}$ is a Bayesian belief function.
Corollary 2. $Bel(N_h)+Bel(N_{\sim h})=1$.
Corollary 3. $Bel(N_c)+Bel(N_d)=1$.

When a symptom is observed, some system states being inconsistent with observations are eliminated. A prior belief function becomes the posterior belief function conditioned on N_d.

$$Bel'(N_h)=Bel(N_h/N_d)=\frac{Bel(N_h\cup N_c)-Bel(N_c)}{1-Bel(N_c)}.$$

From (4), the posterior belief function can also be presented by the prior probabilities,

$$Bel'(N_h)=\frac{P(N_h\cup N_c)-P(N_c)}{1-P(N_c)},$$

and

$$Bel'(N_{\sim h})=\frac{P(N_{\sim h}\cup N_c)-P(N_c)}{1-P(N_c)}.$$

If the conflict set composes of m minimal conflicts, we have

$$N_c=\bigcup_{i=1}^{m} N_{c_i}.$$

Using theorem 4,

$$N_c=\bigcup_{i=1}^{m} N'_{c_i},$$

with $N'_{c_1}=N_{c_1}$, $N'_{c_i}=N_{c_i}\cap\sim N_{c_1}\cap\cdots\cap\sim N_{c_{j\neq i}}$. So

$$P(N_c)=P(N'_{c_1})+\cdots+P(N'_{c_m}). \tag{5}$$

Similarly, we have

$$N_{\sim h} \bigcup N_c = N_{\sim h} \cup (N_c \cap N_h).$$

Theorem 7. If $N_c = \bigcup_{i=1}^{m} N'_{c_i}$, and $N'_{c_i} \cap N'_{c_j} = \Phi$, then $N_c \cap N_h = \bigcup_{i=1}^{m} (N'_{c_i} \cap N_h)$, $(N'_{c_i} \cap N_h) \cap (N'_{c_{j\neq i}} \cap N_h) = \Phi$.

Proof: If $x \in N_h \cap N_{c,}$ then $x \in N_1'$ or … or $x \in N_n'$, and $x \in N_h$, therefore $x \in N_1'$ and $x \in N_h$ or … or $x \in N_n'$ and $x \in N_h$, thus $N_c \cap N_h = \bigcup_{i=1}^{m} (N'_{c_i} \cap N_h)$. If $x \in N_h \cap N'_{c_i}$, then $x \in N'_{c_i}$, $x \notin N'_{c_{j\neq i}}$ $x \notin (N_h \cap N'_{c_{j\neq i}})$. Thus $(N'_{c_i} \cap N_h) \cap (N'_{c_{j\neq i}} \cap N_h) = \Phi$.

By Theorem 7,

$$P(N_h \cap N_c) = P(N_h \cap N'_{c_1}) + \cdots + P(N_h \cap N'_{c_m}).$$

And using theorem 4,

$$\begin{aligned} P(N_{\sim h} \cup N_c) &= P(N_{\sim h}) + P(N_c \cap N_h) \\ &= 1 - P(N_h) + \sum_{i=1}^{m} P(N'_{c_i} \cap N_h). \end{aligned} \tag{6}$$

5. EXAMPLE OF THE BELIEF FUNCTION MODEL OF MODEL-BASED DIAGNOSIS

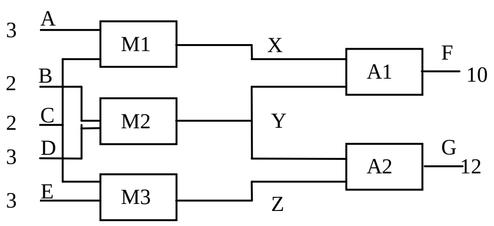

Figure 1: An electrical circuit example

Figure 1 shows a simple electrical circuit comprised of three multipliers (M1, M2, and M3) and two adders (A1 and A2). Suppose that terminals A, B, C, D and E are given the inputs A = 3, B = 2, C = 2, D = 3 and E = 3 respectively. The expected output values at terminals F and G are both 12. However, two outputs F = 10 and G = 12 are observed. The discrepancy between the prediction and observation at F indicates that the circuit is malfunctioning. One or more components must be faulty. The diagnostic task is to locate faulty components in the circuit which has produced the observed output.

In this system, each of the five independent components is assumed to hold two possible statuses: intact and faulty respectively, *e.g.* five state spaces are $M_1=\{m_1, \sim m_1\}$, $M_2=\{m_2, \sim m_2\}$, $M_3=\{m_3, \sim m_3\}$, $A_1=\{a_1, \sim a_1\}$, and $A_2=\{a_2, \sim a_2\}$ respectively. The Cartesian product of them sets up the state space of the system $N=M_1\times M_2\times M_3\times A_1\times A_2$. We make the following assumptions about the prior probabilities distributed to the individual states of these components.

$$p_{\{m_1\}} = 1 - p_{\{\sim m_1\}} = p_{\{m_2\}} = 1 - p_{\{\sim m_2\}} = p_{\{m_3\}} = 1 - p_{\{\sim m_3\}} = 0.94,$$

and

$$p_{\{a_1\}} = 1 - p_{\{\sim a_1\}} = p_{\{a_2\}} = 1 - p_{\{\sim a_2\}} = 0.97\ .$$

Each system state has a prior probability distribution calculated by equation (2). For example, the state $x = \{m_1, m_2, m_3, a_1, \sim a_2\}$ has the prior probability

$$p(x) = p_{\{m_1\}} \times p_{\{m_2\}} \times p_{\{m_3\}} \times p_{\{a_1\}} \times p_{\{\sim a_2\}} = 0.94^3 \times 0.97 \times (1-0.97) = 0.024\,.$$

As two outputs at F and G in Figure 1 are observed, many system states become impossible and must be excluded. The minimal conflict sets are <m_1, m_2, a_1> and <m_1, m_3, a_1, a_2>, the corresponding conflict states comprise the set N_c,

$$\begin{aligned}
N_c &= \{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\cup\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times M_2\} \\
&= \{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\cup\{\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times M_2\} \\
&\quad \cap \sim \{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\} \\
&= \{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\cup\{\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times M_2\} \\
&\quad \cap \sim \{\{\{m_1\}\times M_2\times M_3\times A_1\times A_2\}\cap\{\{\mathrm{m}_2\}\times M_1\times M_3\times A_1\times A_2\} \\
&\quad \cap\{\{\mathrm{a}_1\}\times M_1\times M_2\times M_3\times A_2\}\}\} \\
&= \{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\cup\{\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times M_2\} \\
&\quad \cap\{\sim \{\{m_1\}\times M_2\times M_3\times A_1\times A_2\}\cup \sim \{\{\mathrm{m}_2\}\times M_1\times M_3\times A_1\times A_2\} \\
&\quad \cup \sim \{\{\mathrm{a}_1\}\times M_1\times M_2\times M_3\times A_2\}\}\} \\
&= \{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\cup\{\{\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times M_2\} \\
&\quad \cap\{\{\sim m_1\}\times M_2\times M_3\times A_1\times A_2\}\}\cup\{\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times M_2\} \\
&\quad \cap\{\{\sim m_2\}\times M_1\times M_3\times A_1\times A_2\}\}\cup\{\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times M_2\} \\
&\quad \cap\{\{\sim \mathrm{a}_1\}\times M_1\times M_2\times M_3\times A_2\}\}\} \\
&= \{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\cup\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times\{\sim m_2\}\}.
\end{aligned}$$

The prior probability on N_c is then equal to

$$\begin{aligned}
P(N_c) &= P(\{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}) + P(\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times\{\sim m_2\}\}) \\
&= p_{\{m_1\}}\times p_{\{m_2\}}\times p_{\{a_1\}} + p_{\{m_1\}}\times p_{\{m_3\}}\times p_{\{a_1\}}\times p_{\{a_2\}}\times p_{\{\sim m_2\}} \\
&= 0.94^2\times 0.97 + 0.94^2\times 0.97^2\times(1-0.94) \\
&= 0.907.
\end{aligned}$$

Considering the hypothesis *h1* that M1 is faulty, the corresponding state set

$$N_{h1} = \{\{\sim m_1\}\times M_2\times M_3\times A_1\times A_2\},$$

and the complement set of N_{h1} is $N_{\sim h1}$.

$$\begin{aligned}
N_c \cap N_{h1} &= \{\{\{m_1\}\times\{m_2\}\times\{a_1\}\times M_3\times A_2\}\cup\{\{m_1\}\times\{m_3\}\times\{a_1\}\times\{a_2\}\times\{\sim m_2\}\}\} \\
&\quad \cap\{\{\sim m_1\}\times M_2\times M_3\times A_1\times A_2\} \\
&= \Phi.
\end{aligned}$$

The posterior belief degree on $N_{\sim h1}$ equals to

$$Bel'(N_{\sim h1}\} = \frac{1-P(N_{h1})-P(N_c)}{1-P(N_c)} = \frac{1-0.06-0.907}{1-0.907} = 0.35.$$

Then $Bel'(N_{h1}) = 1 - Bel'(N_{\sim h1}) = 1 - 0.35 = 0.65$.

Similarly, we can compute the posterior beliefs for hypotheses $h2 = \{\sim a_1\}$, $h3 = \{\sim m_2, \sim m_3\}$, and $h4 = \{\sim m_2, \sim a_2\}$, which have been remained being further discriminated to eliminate less likely ones.

$$Bel'(N_{h2}) = 0.32,\ Bel'(N_{\sim h2}) = 0.68;$$

$$Bel'(N_{h3}) = 0.04,\ Bel'(N_{\sim h3})=0.96;$$

$$Bel'(N_{h4}) = 0.02,\ Bel'(N_{\sim h4})=0.98;$$

These results show that the single component Multiplier M1 is faulty is the most likely cause accounting for all of the discrepancies having been observed.

The above example illustrates how beliefs can help model-based diagnosis to locate precisely the faulty component(s) before a further measurement is carried out. The approach is suitable for either a system with only a single fault or a system with multiple faults.

6. CONCLUSIONS AND DISCUSSIONS

In this paper, we proposed a belief function model of model-based diagnosis. The beliefs distributed to hypotheses are used to eliminate hypotheses one step before any new measurements are made. An efficient process to compute and update the beliefs when new symptoms have been observed has also been presented after carrying out a full exploration of the properties of the conflict set, candidate set and hypothesis set.

When multiple faults are diagnosed, further tests need to be carried out to eliminate candidates which contribute to the symptoms. Our immediate future work is to design algorithms that can eliminate less possible candidates that may have contributed to multiple faults.

ACKNOWLEDGEMENTS

This research was firstly carried out when the first author was working on the JIGSAW research programme supported by the Nortel Networks at Monkstown and IRTU. Now the author expands this research to automated software testing under the new project at the Centre for Software Processing Technology.

REFERENCES

[1] Raymond Reiter, 1987, "A Theory of Diagnosis from First Principles", Artificial Intelligence, Vol.32, pp.57-95.
[2] Johan de Kleer, 1986, "Problem solving with the ATMS", Artificial Intelligence, Vol.28, pp.197-224.
[3] Johan de Kleer, 1990, "Using Crude Probability Estimates to Guide Diagnosis", Artificial Intelligence, Vol.45, pp.381-391.
[4] A. P. Dempster, 1967, "Upper and Lower Probabilities Induced by a Multivalued Mapping", Annals of Mathematical Statistics, Vol.38, pp.325-339.
[5] Glenn Shafer, 1976, "A Mathematical Theory of evidence", Princeton University Press, Chichester, West Sussex.
[6] Johan de Kleer and Brian C. Williams, 1987, "Diagnosing Multiple Faults", Artificial Intelligence, Vol.32, pp.97-130.
[7] Joseph Y. Halpern and Ronald Fagin, 1990, "Two views of belief: Belief as generalized probability and belief as evidence", AAAI, pp.112-119.
[8] J.Kohlas; B. Anrig; R. Haenni; P. A. Monney, 1998, "Model-based diagnostics and probabilistic assumption-based reasoning", Artificial Intelligence, Vol.104, pp.71-106.
[9] Laurence P. Maher, 1968, "Finite Sets: Theory, Counting, and Applications", Charles E. Merrill Publishing Company.
[10] Meyer Dwass, 1970, "Probability and Statistics", W. A. Benjamin, Inc., New York.