A Characteristic Function Approach to Inconsistency Measures for Knowledge Bases

Jianbing Ma, Weiru Liu, and Paul Miller

School of Electronics, Electrical Engineering and Computer Science, Queen's University Belfast, Belfast BT7 1NN, UK
{jma03,w.liu,p.miller}@qub.ac.uk

Abstract. Knowledge is an important component in many intelligent systems. Since items of knowledge in a knowledge base can be conflicting, especially if there are multiple sources contributing to the knowledge in this base, significant research efforts have been made on developing inconsistency measures for knowledge bases and on developing merging approaches. Most of these efforts start with flat knowledge bases. However, in many real-world applications, items of knowledge are not perceived with equal importance, rather, weights (which can be used to indicate the importance or priority) are associated with items of knowledge. Therefore, measuring the inconsistency of a knowledge base with weighted formulae as well as their merging is an important but difficult task. In this paper, we derive a numerical characteristic function from each knowledge base with weighted formulae, based on the Dempster-Shafer theory of evidence. Using these functions, we are able to measure the inconsistency of the knowledge base in a convenient and rational way, and are able to merge multiple knowledge bases with weighted formulae, even if knowledge in these bases may be inconsistent. Furthermore, by examining whether multiple knowledge bases are dependent or independent, they can be combined in different ways using their characteristic functions, which cannot be handled (or at least have never been considered) in classic knowledge based merging approaches in the literature.

Keywords Knowledge Bases, Characteristic Function, Inconsistency Measure, Merging, Evidence Theory.

1 Introduction

Logic based knowledge representation is used in many cases, such as software requirements [18], expert systems [20], belief merging [9]. In most of the applications, logic based knowledge bases (KB) are flat, that is all formulae in the base are equally important. However, in some applications, such as requirement engineering [19], some formulae can be more important than others. So ranked or stratified knowledge bases are commonly deployed. The importance of a formula can also be modelled by attaching a numerical value to the formula, which in some case is explained as a *weight*.

When a numerical value is attached to a logical formula, this value can be explained in many different ways according to the semantics of this value. Some typical explanations are belief degrees, preference degrees, truth degrees, trust degrees. In this paper, we consider a numerical value as a weight indicating the importance (or priority) of this formulae w.r.t other formulae in the same knowledge base and we study both flat KBs in which all the logical formulae are viewed as equally important and weighted knowledge bases in which each formula is associated with a weight.

It should be noted that the meaning of a knowledge base having weights attached to them is different from the *weighted knowledge bases* discussed in [10], in which a weighted knowledge base means that a knowledge base as a whole is attached with a weight representing the relative degree of importance (or reliability) of the source from which the knowledge base is derived.

There are two frequently studied topics for knowledge bases. One is on measuring the inconsistency of a knowledge base (e.g., [18, 8]) and the other is the merging of multiple knowledge bases e.g., [9, 12, 13]). Most of these approaches deploy logic-based formalisms, even when priorities or degrees of certainty are involved. An interesting phenomenon is that although these two issues are closely related (if we view the aim of merging is to obtain a consistent knowledge base), their solutions usually follow different paths. Therefore, a natural question is: can we develop an underlying formalism which can handle both of the issues simultaneously?

Another potential problem on merging multiple knowledge bases is that, so far in the literature, merging of knowledge bases does not really consider any possible dependency relationship among knowledge bases, which subsequently causes the difficulty of justifying a merging result. For example, if two experts have provided their knowledge in terms of weighted formulae, and one expert has been heavily influenced by another, then their knowledge bases are not totally independent. That is if K_1, K_2 are the two knowledge bases from these two experts, and they are represented as $K_1 = K_2 = \{(\alpha, 0.8), (\beta, 0.2)\}$, then the merging result K should be identical to either of them, if K_1 and K_2 are dependent. On the other hand, if K_1 and K_2 are independent, then we should expect that the weight on α being increased and the weight on β being decreased. Formally, K should be $\{(\alpha, x), (\beta, y)\}$ such that $x > 0.8$ and $y < 0.2$. That is, whether some knowledge bases have a dependent relationship should influence how merging should be carried out. However, this issue has not been explicitly discussed in classical logic based approaches for merging knowledge bases in the literature. Thus, another question could be asked here is: can we reflect the information on dependency relationship among knowledge bases when performing merging?

In this paper, we provide positive answers for both questions mentioned above. We propose a *characteristic function* for a knowledge base with formulae having weights, and a flat knowledge base is treated as a special case where all formulae having the same weight. The characteristic function entails all the information of a knowledge base provides and hence can be used to measure the inconsistency of the knowledge base and to handle the merging of multiple knowledge bases. Characteristic functions are defined in the form of basic probability assignments in the Dempster-Shafer (DS) theory [3, 4, 21, **?**,**?**].

Example 1 *In [16, 17, 15], an intelligent surveillance system was designed and developed. In this project, interested events are recognized from analyzing data coming from different sources (e.g., cameras) and these event descriptions usually contain uncertain information, such as, an gender-profile event describes a passenger as a Male with 70%*

probability and the rest is unknown. Hence DS theory is introduced to model the uncertainty implied in the events and combination approaches are applied to combine gender recognition events descriptions from multiple sources. Furthermore, a knowledge base is developed which contain inference rules for inferring high-level events from a set of elementary recognized events.

Rules are elicited from domain experts. Each expert provides its knowledge containing multiple rules, each of which is attached with a weight indicating its importance. Examples of simple rules are as follows:

Let a *denote* a passenger A is a male*,* b *denote* A is shouting*,* c *denote* A is dangerous*, and* d *denote* A is an old lady*. Then an expert's knowledge could contain* $K = \{(a \wedge b \to c, 0.8), (d \wedge b \to \neg c, 0.2)\}$ *which is semantically explained as* if a passenger A is a Male and A is shouting, then passenger A is dangerous*,* if a passenger A is an old lady and A is shouting, then passenger A is not dangerous.

Here, numerical values 0.8 and 0.2 are the weights of these two rules, which says that a rule indicating a dangerous event is more important than otherwise[1]*.*

When different experts provide their knowledge in terms of such knowledge base with weighted formulae, we need to merge them. This can be achieved by using their corresponding characteristic functions.

This characteristic function approach also absorbs some nutrients from papers such as [1, 13, 14], etc., in which methods of inducing probability measures from knowledge bases are studied that demonstrates the demand and usefulness of quantitative methods on managing knowledge bases.

The main contribution of this paper is as follows:

- From a knowledge base with weighted formulae, a unique basic belief assignment (bba) can be recovered. To the best of our knowledge, there is no paper having emphasized this point.
- We show that the associated bba could be used to measure the inconsistency of the knowledge base and to merge multiple knowledge bases in a quantitative way that is beyond the usual approaches in the knowledge base inconsistency measure / merging fields.
- We show that merging of knowledge bases can take into account dependencies between knowledge bases using this approach.

The rest of the paper is organized as follows. In Section 2, we recall some basic concepts and notations of propositional language, knowledge bases and the evidence theory. In Section 3, we define the characteristic function of a knowledge base. In Section 4, we provide an inconsistency measure of a knowledge base and show some rational properties of this inconsistency measure. In Section 5, we discuss various merging methods of knowledge bases using their characteristic functions. Finally, in Section 6, we conclude the paper.

[1] It should be noted that this weight about the importance of the rule should not be confused with a statistical value (such as 0.8) showing the likelihood of how dangerous a shouting male passenger could be. The later is explained as that when a male passenger is shouting, there is 80% chance this will lead to a dangerous consequence.

2 Preliminaries

2.1 Knowledge Bases

Here we consider a propositional language $\mathcal{L}_{\mathcal{P}}$ defined from a finite set $\mathcal{P}$ of propositional atoms, which are denoted by p, q, r etc (possibly with sub or superscripts). A proposition ϕ is constructed by atoms with logical connectives $\neg, \wedge, \vee$ in the usual way. An interpretation w (or possible world) is a function that maps $\mathcal{P}$ onto the set $\{0, 1\}$. The set of all possible interpretations on $\mathcal{P}$ is denoted as W. Function $w \in W$ can be extended to any propositional sentence in $\mathcal{L}_{\mathcal{P}}$ in the usual way, $w : \mathcal{L}_{\mathcal{P}} \to \{0, 1\}$. An interpretation w is a model of (or satisfies) ϕ iff $w(\phi) = 1$, denoted as $w \models \phi$. We use $Mod(\phi)$ to denote the set of models for ϕ.

For convenience, let $form(\{w_1, \cdots, w_n\})$ be the formula whose models are exactly $w_1, \cdots, w_n$, and also $form(A)$ denote a formula μ such that $Mod(\mu) = A$.

A *flat knowledge base* K is a finite set of propositional formulas. K is consistent iff there is at least one interpretation that satisfies all the propositional formulas in K.

A *weighted knowledge base* K is a finite set of propositional formulas, each of which has a numerical value, called weight, attached to it i.e., $\{(\mu_1, x_1), \cdots, (\mu_n, x_n)\}$ where $\forall i, 0 < x_i \leq 1$. In fact, the requirement $x_i \in [0, 1]$ can be relaxed to allow x_i taking any positive numerical value. In that case, a normalization step will reduce each x_i to a value within $[0, 1]$. Obviously, if all x_is are equivalent, then a weighted knowledge base is reduced to a flat knowledge base. Conversely, a flat knowledge base can be seen as a weighted knowledge base, e.g., $K = \{(\mu_1, \frac{1}{n}), \cdots, (\mu_n, \frac{1}{n})\}$. Therefore, for convenience, in the rest of the paper, we refer to all knowledge bases as weighted knowledge bases.

For a weighted knowledge base $K = \{(\mu_1, x_1), \cdots, (\mu_n, x_n)\}$, we let $\hat{K} = \{\mu_1, \cdots, \mu_n\}$ be its corresponding flat knowledge base in which the weights of all the formulae of K are removed. For a formula μ, we write $\mu \in K$ if and only if $\mu \in \hat{K}$. In addition, K is consistent if and if $\hat{K}$ is consistent.

If a classical knowledge base $\hat{K}$ is inconsistent, then we can define its minimal inconsistent subsets as follows [2, 7]:

$$MI(\hat{K}) = \{\hat{K}' \subseteq \hat{K} | \hat{K}' \vdash \perp \text{ and } \forall \hat{K}'' \subset \hat{K}', \hat{K}'' \nvdash \perp\}.$$

A ***free formula*** of a knowledge base $\hat{K}$ is a formula of $\hat{K}$ that does not belong to any minimal inconsistent subset of the knowledge base $\hat{K}$ [2, 7]. A free formula in a weighted knowledge base K is defined as a free formula of $\hat{K}$.

2.2 Evidence Theory

We also recall some basic concepts of Dempster-Shafer's theory of evidence.

Let Ω be a finite set called the frame of discernment (or simply frame). In this paper, we denote $\Omega = \{w_1, \ldots, w_n\}$.

Definition 1 *A basic belief assignment (bba for short) is a mapping* $m : 2^{\Omega} \to [0, 1]$ *such that* $\sum_{A \subseteq \Omega} m(A) = 1$.

A bba m is also called a mass function when $m(\emptyset) = 0$ is required. A vacuous bba m is such that $m(\Omega) = 1$.

If $m(A) > 0$, then A is called a focal element of m. Let $\mathscr{F}(m)$ denote the set of the focal elements of m. That is, if A is a focal elements of m, then $A \in \mathscr{F}(m)$.

Let $\oplus$ be the conjunctive combination operator (or Smets' operator [22]) for any two bbas m, m' over Ω such that

$$(m \oplus m')(C) = \sum_{A \subseteq \Omega, B \subseteq \Omega, A \cap B = C} m(A)m'(B), \forall\, C \subseteq \Omega.$$

Particularly, we have:

$$(m \oplus m')(\emptyset) = \sum_{A \subseteq \Omega, B \subseteq \Omega, A \cap B = \emptyset} m(A)m'(B). \tag{1}$$

A simple bba m such that $m(A) = x, m(\Omega) = 1 - x$ for some $A \neq \Omega$ is denoted as A^x. The vacuous bba can thus be denoted as A^0 for any $A \subset \Omega$. By abuse of notations, we also use μ^x to denote the simple bba A^x where $A = Mod(\mu)$. Note that this notation, i.e., A^x, is a different from the one defined in [5] such that A^x in our paper should be denoted as A^{1-x} based on explanations in [5].

3 Characteristic Functions

In this section, we define characteristic functions for knowledge bases.

A first and direct thought for characteristic function of a knowledge base $K = \{(\mu_1, x_1), \cdots, (\mu_n, x_i)\}$ is to define it as follows.

$$m'_K(Mod(\mu_i)) = x_i, 1 \leq i \leq n.$$

But this characteristic function definition brings many problems. For example, it is difficult to use this characteristic function to measure inconsistency of a single knowledge base. Since the usual way of defining inconsistency of a mass function is its *empty mass*. Hence, a simple definition of the inconsistency of K could be:

$$Inc'(K) = (m'_K \oplus m'_K)(\emptyset).$$

That is, the internal inconsistency of K is the empty mass when K interacts with itself, i.e., $m'_K \oplus m'_K$.

But it does not give reasonable results. For instance, if m'_K is such that $m'_K(\{w_1, w_2\}) = m'_K(\{w_1, w_3\}) = m'_K(\{w_2, w_3\}) = \frac{1}{3}$, then we have $(m'_K \oplus m'_K)(\emptyset) = 0$. But the corresponding knowledge base $\hat{K} = \{(form(\{w_1, w_2\}), \frac{1}{3}), (form(\{w_1, w_3\}), \frac{1}{3}), (form(\{w_2, w_3\}), \frac{1}{3})\}$ is not consistent since $form(\{w_1, w_2\}) \wedge form(\{w_1, w_3\}) \wedge form(\{w_2, w_3\}) \vdash \bot$.

Therefore, we define our characteristic function for a weighted knowledge base as follows.

Definition 2 *For any weighted knowledge base $K = \{(\mu_1, x_1), \cdots, (\mu_n, x_i)\}$, its corresponding characteristic function is m_K such that $m_K = \oplus_{i=1}^n \mu_i^{x_i}$.*

This characteristic function makes use of both the formulae (μ_i) in K and their weights (x_i), which produces a bba that is a kind of characterization of K. More precisely, the defined characteristic function m_K is unique for K. That is, if K_1 and K_2 are logically different[2], then m_{K_1} should also be different from m_{K_2}. This is ensured by the following result [5]:

A bba m such that $m(\Omega) > 0$ can be uniquely decomposed into the following form:

$$m = \oplus_{\phi:Mod(\phi)\subset\Omega}\phi^{x_\phi}, x_\phi \in [0, 1]. \tag{2}$$

That is, if $m_{K_1} = m_{K_2}$, then based on Equation 2, they have the same decomposition and hence the same knowledge base (in a sense that vacuous information is ignored). Equation 2 also demonstrates that the bba m_K encodes all the information contained in K. In fact, we can easily recover K from m_K with a few steps. Since recovering K is not the main focus of this paper, and due to the space limitation, the details of how to recover μ_i and x_i from m_K is omitted here. Interested readers could be refer to [5].

A simple result about this characteristic function is shown as follows.

Proposition 1 *For any weighted knowledge base K and any x such that $0 < x \leq 1$, we have $m_K = m_{K \bigcup \{\top, x\}}$.*

That is, vacuous information does not change the characteristic function. This is an intuitive result and it does not contradict the former statement that m_K induces a unique K, since vacuous information in K could somehow be ignored. For example, if someone tells you: tomorrow will be either sunny or not sunny. Obviously this piece of vacuous information could be ignored.

4 Inconsistency Measure

In this section, we use the characteristic function of a knowledge base to measure its inconsistency. In addition, we prove that this inconsistency measure satisfies a set of rational properties proposed in [7].

Definition 3 *For any weighted knowledge base K, the inconsistency measure of K is defined as:*

$$Inc(K) = m_K(\emptyset).$$

Taking $m_K(\emptyset)$ as the inconsistency measure for K is very natural since in DS theory, $m_K(\emptyset)$ is a largely used to measure the degree of conflict between beliefs of agents[3].

Now we show that this definition of inconsistency satisfies some intuitive properties. In [7], a set of properties that an inconsistency measure I for a knowledge base shall have is proposed as follows.

[2] That is, two different but logically equivalent formulas are considered equivalent here.

[3] While in several papers, most notably in [11], it is argued that $m_K(\emptyset)$ is not enough for an inconsistency measure for bbas. However, in most applications, $m_K(\emptyset)$ is still being used to measure the inconsistency between bbas.

Definition 4 *([7]) An inconsistency measure I is called a basic inconsistent measure if it satisfies the following properties:*

for any flat knowledge bases K, K' and any two formulae α, β:

Consistency $I(K) = 0$ *iff K is consistent*
Normalization $0 \leq I(K) \leq 1$
Monotony $I(K \bigcup K') \geq I(K)$
Free Formula Independence *If α is a free formula of $K \bigcup \{\alpha\}$, then* $I(K \bigcup \{\alpha\}) = I(K)$
Dominance *If $\alpha \vdash \beta$ and $\alpha \not\vdash \bot$, then* $I(K \bigcup \{\alpha\}) \geq I(K \bigcup \{\beta\})$

For weighted knowledge bases, the above definition should be adapted to:

Definition 5 *An inconsistency measure I is called* a basic inconsistent measure for weighted knowledge bases *if it satisfies the following properties:*

for any weighted knowledge bases K, K', any two formulae α, β and any real x, $0 < x \leq 1$:

Consistency $Inc(K) = 0$ *iff K is consistent.*
Normalization $0 \leq Inc(K) \leq 1$.
Monotony $Inc(K \bigcup K') \geq Inc(K)$.
Free Formula Independence *If α is a free formula of $K \bigcup \{(\alpha, x)\}$, then* $Inc(K \bigcup \{(\alpha, x)\}) = Inc(K)$.
Dominance *If $\alpha \vdash \beta$ and $\alpha \not\vdash \bot$, then* $Inc(K \bigcup \{(\alpha, x)\}) \geq Inc(K \bigcup \{(\beta, x)\})$.

We prove that our inconsistency measure satisfies all the above properties. In addition, we show that our inconsistency measure satisfies a *Strong Free Formula Independence* property as follows.

Strong Free Formula Independence $Inc(K \bigcup \{(\alpha, x)\}) = Inc(K)$ if and only if α is a free formula of $K \bigcup \{(\alpha, x)\}$.

Proposition 2 *For any weighted knowledge base K, $Inc(K)$ is a basic inconsistency measure. In addition, $Inc(K)$ satisfies the* Free Formula Independence *property.*

Proof of Proposition 2:

Consistency $Inc(K) = 0$ iff K is consistent.
$Inc(K) = 0$ iff $\forall \mu_i \in K$, $\bigcap_{i=1}^{n} \mu_i \not\vdash \bot$ iff K is consistent.
Normalization $0 \leq Inc(K) \leq 1$.
Obvious.
Monotony $Inc(K \bigcup K') \geq Inc(K)$.
We show that for any formula α and $0 < x \leq 1$, $Inc(K \bigcup \{(\alpha, x)\}) \geq Inc(K)$. In fact, we have

$$\begin{aligned} Inc(K \bigcup \{(\alpha, x)\}) &= m_{(K \bigcup \{(\alpha,x)\})}(\emptyset) \\ &= m_K(\emptyset) + \sum_{A \in \mathscr{F}(m_K), A \neq \emptyset, A \cap Mod(\alpha) = \emptyset} m_K(A) \times x \\ &\geq m_K(\emptyset) \\ &= Inc(K), \end{aligned}$$

hence without loss of generality, assume $K' = \{(\alpha_1, x_1), \cdots, (\alpha_n, x_n)\}$, we then have

$$\begin{aligned} Inc(K) &\leq Inc(K \bigcup \{(\alpha_1, x_1)\}) \\ &\leq \cdots \\ &\leq Inc(K \bigcup K') \end{aligned}$$

Free Formula Independence If α is a free formula of $K \bigcup \{(\alpha, x)\}$, then $Inc(K \bigcup \{(\alpha, x)\}) = Inc(K)$.

If α is a free formula of $K \bigcup \{(\alpha, x)\}$, then there does not exist $A \in \mathscr{F}(m_K)$ and $A \neq \emptyset$, s.t., $A \cap Mod(\alpha) = \emptyset$, hence we have

$$\begin{aligned} Inc(K \bigcup \{(\alpha, x)\}) &= m_K(\emptyset) + \sum_{A \in \mathscr{F}(m_K), A \neq \emptyset, A \cap Mod(\alpha) = \emptyset} m_K(A) \times x \\ &= m_K(\emptyset) \\ &= Inc(K). \end{aligned}$$

Dominance If $\alpha \vdash \beta$ and $\alpha \not\vdash \perp$, then $Inc(K \bigcup \{(\alpha, x)\}) \geq Inc(K \bigcup \{(\beta, x)\})$.

It is straightforward from a simple fact that for any $A \in \mathscr{F}(m_K)$ and $A \neq \emptyset$, if $A \cap Mod(\beta) = \emptyset$, then $A \cap Mod(\alpha) = \emptyset$, hence

$$\begin{aligned} Inc(K \bigcup \{(\beta, x)\}) &= m_K(\emptyset) + \sum_{A \in \mathscr{F}(m_K), A \neq \emptyset, A \cap Mod(\beta) = \emptyset} m_K(A) \times x \\ &\leq m_K(\emptyset) + \sum_{A \in \mathscr{F}(m_K), A \neq \emptyset, A \cap Mod(\alpha) = \emptyset} m_K(A) \times x \\ &= Inc(K \bigcup \{(\alpha, x)\}) \end{aligned}$$

Strong Free Formula Independence $Inc(K \bigcup \{(\alpha, x)\}) = Inc(K)$ if and only if α is a free formula of $K \bigcup \{(\alpha, x)\}$.

It is obvious from the prove of **Free Formula Independence** property. □

It is worth pointing out that our inconsistency measure can be naturally used to deal with weighted knowledge bases in addition to flat knowledge bases, while the existing inconsistency measures based on minimal inconsistency subsets or from a classic logic-based approach are incapable of tackling with weighted knowledge bases.

Example 2 *Let* $K_1 = \{(\alpha, 0.8), (\alpha \vee \beta, 0.2)\}$ *and* $K_2 = \{(\alpha, 0.6), (\beta, 0.3), (\alpha \vee \beta, 0.1)\}$. *Then the characteristic function of* K_1*, i.e.,* m_{K_1}*, is such that*

$$m_{K_1}(\alpha) = 0.8, m_{K_1}(\alpha \vee \beta) = 0.2.$$

Hence it is easy to see that $Inc(K_1) = 0$.

Then the characteristic function of K_2*, i.e.,* m_{K_2}*, is such that*

$$m_{K_2}(\emptyset) = 0.18, m_{K_2}(\alpha) = 0.42, m_{K_2}(\beta) = 0.12, m_{K_2}(\alpha \vee \beta) = 0.28.$$

Hence we easily get $Inc(K_2) = 0.18$.

5 Merging

In this section, we discuss the merging of weighted knowledge bases using their characteristic functions. Since characteristic functions are in the form of bbas, the merging methods are hence based on combination rules of bbas in DS theory. There are many different combination rules for bbas, which are similar to the many different merging strategies for knowledge bases. For simplicity, here we only mention Dempster's rule and Didier & Prade's hybrid rule.

For convenience and convention, if there is no confusion, in the following we may use both sets and formulae, e.g., $m(\emptyset) = 0.4, m(\phi) = 0.6$, etc., just be noted that any propositional formulae used in these situations, like ϕ, are in fact standing for $Mod(\phi)$. Conversely, when we write $K = \{(A, 0.5), (B, 0.3)\}$, it simply means $K = \{(form(A), 0.5), (form(B), 0.3)\}$. This short-hand notation is simply for making the mathematical formulas shorter and does not suggest any technical changes.

The subsequent definitions use combination rules of bbas, but as our aim is to merge knowledge bases, we will call them merging methods. Therefore, in the following, each definition defines a merging method for merging weighted knowledge bases using their characteristic functions. Also note that the merging methods only give the characteristic function for the merged knowledge base since from this function, its corresponding knowledge base can be easily induced (Equation (2), cf. [5] for details).

Definition 6 *(Dempster's Merging, [3, 4, 21]) Let K_1, K_2 be two knowledge bases and m_{K_1}, m_{K_2} be their characteristic functions, respectively, then the characteristic function $m_{K_{12}}$ of the merged knowledge base by Dempster's combination rule is such that:*

$$m_{K_{12}}^{Dem}(A) = \frac{\sum_{B,C\subseteq\Omega,B\cap C=A} m_1(B)m_2(C)}{1 - \sum_{B,C\subseteq\Omega,B\cap C=\emptyset} m_1(B)m_2(C)}, \forall A \subseteq \Omega, A \neq \emptyset,$$
$$m_{K_{12}}^{Dem}(\emptyset) = 0.$$

Definition 7 *(Dubois and Prade's Merging, [6]) Let K_1, K_2 be two knowledge bases and m_{K_1}, m_{K_2} be their characteristic functions, respectively, then the characteristic function $m_{K_{12}}$ of the merged knowledge base by DP's combination rule is such that:*

$$m_{DP}(\emptyset) = 0$$
$$m_{DP}(A) = \sum_{B,C\subseteq\Omega,B\cap C=A} m_1(B)m_2(C) + \sum_{B,C\subseteq\Omega,B\cup C=A,B\cap C=\emptyset} m_1(B)m_2(C), \forall A \subseteq \Omega, A \neq \emptyset$$

In Dempster's merging, weights of conflicting formulae are proportionally distributed to formulae resulted from intersection of non-conflicting formulae. Instead, in Dubois & Prade's Merging, weights of conflicting formulae are added to the disjunction of the conflicting formulae.

Example 3 *(Example 2 Continued) In Example 2, we have that the characteristic function of K_1, i.e., m_{K_1}, is such that $m_{K_1}(\alpha) = 0.8, m_{K_1}(\alpha \vee \beta) = 0.2$, and the characteristic function of K_2, i.e., m_{K_2}, is such that $m_{K_2}(\emptyset) = 0.18, m_{K_2}(\alpha) = 0.42, m_{K_2}(\beta) = 0.12, m_{K_2}(\alpha \vee \beta) = 0.28$, then we can get the following merging results using the above merging methods.*

Dempster's Merging $m_{K_{12}}^{Dem}$ *is such that*

$$m_{K_{12}}^{Dem}(\alpha) = 0.89, m_{K_{12}}^{Dem}(\beta) = 0.03, m_{K_{12}}^{Dem}(\alpha \vee \beta) = 0.08$$

DP's Merging $m_{K_{12}}^{DP}$ *is such that*

$$m_{K_{12}}^{DP}(\alpha) = 0.788, m_{K_{12}}^{DP}(\beta) = 0.024, m_{K_{12}}^{DP}(\alpha \vee \beta) = 0.188$$

Note that the above merging methods are not idempotent but have a reinforcement effect. That is, in general, we do not have $\Delta(K, K) = K$ when Δ is a merging operator defined by one of the above merging methods. Reinforcement merging is rational when the knowledge bases to be merged are from distinct sources. However, for knowledge bases from nondistinct sources (i.e., sources providing possibly overlapping knowledge [5]), we intuitively require the merging to be idempotent. To the best of our knowledge, we do not see any idempotent merging methods for knowledge bases in the literature, here we provide an idempotent merging method based on the cautious rule of combination introduced in [5].

Definition 8 *(Denœux's Cautious Merging) Let K_1, K_2 be two knowledge bases and m_{K_1}, m_{K_2} be their characteristic functions, s.t., $m_{K_1} = \oplus_{A \subset \Omega} A^{x_A^1}$ and $m_{K_2} = \oplus_{A \subset \Omega} A^{x_A^2}$, respectively, then the characteristic function $m_{K_{12}}$ of the merged knowledge base by Denœux's cautious combination rule is such that (again notice that our A^{x_A} setting is different from Denœux's):*

$$m_{K_{12}}^{Den} = \oplus_{A \subset \Omega} A^{max(x_A^1, x_A^2)}, \forall A \subset \Omega.$$

Example 4 *(Example 2 Continued) In Example 2, the characteristic function of K_1, i.e., m_{K_1}, is such that $m_{K_1}(\alpha) = 0.8, m_{K_1}(\alpha \vee \beta) = 0.2$, and the characteristic function of K_2, i.e., m_{K_2}, is such that $m_{K_2}(\emptyset) = 0.18, m_{K_2}(\alpha) = 0.42, m_{K_2}(\beta) = 0.12, m_{K_2}(\alpha \vee \beta) = 0.28$, then the characteristic function of the merging result using Denœux's merging method, i.e., $m_{K_{12}}^{Den}$, is as follows.*

$$m_{K_{12}}^{Den}(\emptyset) = 0.24, m_{K_{12}}^{Den}(\alpha) = 0.56, m_{K_{12}}^{Den}(\beta) = 0.06, m_{K_{12}}^{Den}(\alpha \vee \beta) = 0.14.$$

From the characteristic functions, the corresponding knowledge base for $m_{K_{12}}^{Den}$ is:

$$K^{Den} = \{(\alpha, 0.8), (\beta, 0.3)\}.$$

For Denœux's merging method, we have the following result.

Proposition 3 *Let two knowledge bases K_1, K_2 be $K_1 = \{(\mu_1, x_1), \cdots, (\mu_n, x_n)\}$ and $K_2 = \{(\phi_1, y_1), \cdots, (\phi_m, y_m)\}$, then the merging result of K_1 and K_2 using Denœux's merging method is $K_{12} = \{(\psi_1, z_1), \cdots, (\psi_t, z_t)\}$ such that K_{12} is a subset of $K_1 \bigcup K_2$ satisfying the following conditions:*

- *if $\mu_i \equiv \top$, $1 \leq i \leq n$, then $(\mu_i, x_i) \notin K_{12}$; if $\phi_j \equiv \top$, $1 \leq j \leq m$, then $(\phi_j, y_j) \notin K_{12}$,*

– *if $\mu_i \equiv \phi_j$, $1 \leq i \leq n, 1 \leq j \leq m$, and $x_i > y_j$ (resp. $y_j > x_i$), then $(\phi_j, y_j) \notin K_{12}$ (resp. $(\mu_i, x_j) \notin K_{12}$),*
– *all other elements of K_1 or K_2 are in K_{12}.*

Proof of Proposition 3: From $K_1 = \{(\mu_1, x_1), \cdots, (\mu_n, x_n)\}$ and $K_2 = \{(\phi_1, y_1), \cdots, (\phi_m, y_m)\}$, we get $m^{Den}_{K_1} = \oplus^n_{i=1}\mu_i^{x_i}$ and $m^{Den}_{K_2} = \oplus^m_{j=1}\phi_j^{y_j}$.

Let $\{\phi_{j_1}, \cdots, \phi_{j_a}\}$ be the set of all formulae each of which is in K_2 but not in K_1, then we have $m^{Den}_{K_1} = \oplus^n_{i=1}\mu_i^{x_i} \oplus \phi^0_{j_1} \oplus \cdots \oplus \phi^0_{j_a}$.

Similarly, let $\{\mu_{i_1}, \cdots, \mu_{i_b}\}$ be the set of all formulae each of which is in K_1 but not in K_2, we have $m^{Den}_{K_2} = \oplus^m_{j=1}\phi_j^{y_j} \oplus \mu^0_{i_1} \oplus \cdots \oplus \mu^0_{i_b}$.

Hence from Definition 8, it is straightforward to see that the merged characteristic function corresponds to the knowledge base K_{12} which is exactly the same as stated in Proposition 3. $\Box$

Proposition 3 makes it convenient to solve the merging of nondistinct knowledge bases. For instance, from Proposition 3, it is easy to obtain $K^{Den} = \{(\alpha, 0.8), (\beta, 0.3)\}$ from $K_1 = \{(\alpha, 0.8), (\alpha \vee \beta, 0.2)\}$ and $K_2 = \{(\alpha, 0.6), (\beta, 0.3), (\alpha \vee \beta, 0.1)\}$.

In the literature, a basic assumption for knowledge base merging is that the knowledge bases to be merged should be consistent. This assumption is often not applicable in practice, as argued in many research work discussing the inconsistency of a knowledge base (e.g., [18, 7, 8]). An obvious advantage of our merging methods is that they do not require this assumption. That is, even for inconsistent knowledge bases, it is still possible to merge them and obtain rational fusion results. The second advantage is that we can deal with knowledge bases from nondistinct sources. Usually logic-based merging methods do not consider whether the knowledge bases to be combined are from distinct sources. In this paper, however, if the information sources are known to be distinct, then Dempster's merging method, Smets's, Yager's, or Dubois and Prade's merging method can be chosen, whilst if the sources are known to be nondistinct, then Denœux's merging method can be selected. This differentiation of merging methods based on dependency relationship among knowledge bases is obviously more suitable. Of course, proper methods should be developed to judge whether two knowledge bases are dependent or not, but this topic is beyond the scope of this paper.

6 Conclusion

In this paper, we introduced a bba based characteristic function for any weighted knowledge base which take flat knowledge bases as a special case. We then used the characteristic function to measure the inconsistency of the knowledge base and proved that this inconsistency measure follows a set of rational properties. We also deployed the characteristic functions to merge multiple knowledge bases. Different merging methods were provided corresponding to different combination rules of bbas. These merging methods could provide some advantages than the existing merging methods, e.g., the ability to merge inconsistent knowledge bases, the use of distinctness information of knowledge sources, etc.

An obvious future work is to apply these methods in intelligent surveillance applications. In addition, extending this approach to stratified/prioritized/ranked knowledge bases is also an interesting topic with the help of the non-Archimedean infinitesimals [13]. Furthermore, providing comparisons with related works, e.g., our inconsistency measure vs. inconsistency measures in [7]; our merging methods vs. the existing merging methods and merging postulates [9], etc., is a promising issue.

References

1. F. Bacchus, A. Grove, J. Halpern, and D. Koller, 'From statistical knowledge bases to degrees of belief', *Artificial Intelligence*, **87(1-2)**, 75–143, (1996).
2. S. Benferhat, D. Dubois, and H. Prade, 'Representing default rules in possibilistic logic', in *Procs. of KR*, pp. 673–684, (1992).
3. A. P. Dempster, 'Upper and lower probabilities induced by a multivalued mapping', *The Annals of Statistics*, **28**, 325–339, (1967).
4. A. P. Dempster, 'A generalization of bayesian inference', *J. Roy. Statist. Soc.*, **30, Series B**, 205–247, (1968).
5. Thierry Denœux, 'Conjunctive and disjunctive combination of belief functions induced by nondistinct bodies of evidence', *Artifical Intelligence*, **172**(2-3), 234–264, (2008).
6. D. Dubois and H. Prade, 'Representation and combination of uncertainty with belief functions and possibility measures', *Computational Intelligence*, **4**, 244–264, (1988).
7. A. Hunter and S. Konieczny, 'Shapley inconsistency values', in *Procs. of KR'06*, pp. 249–259, (2006).
8. A. Hunter and S. Konieczny, 'Measuring inconsistency through minimal inconsistent sets', in *Procs. of KR*, pp. 358–366, (2008).
9. S. Konieczny and R. Pino-Pérez, 'On the logic of merging', in *KR'98, Principles of Knowledge Representation and Reasoning*, eds., A. G. Cohn, L. Schubert, and S. C. Shapiro, pp. 488–498, San Francisco, California, (1998). Morgan Kaufmann.
10. J. Lin, 'Integration of weighted knowledge bases', *Artif. Intel.*, **83(2)**, 363–378, (1996).
11. W. Liu, 'Analyzing the degree of conflict among belief functions', *Artificial Intelligence*, **170**, 909–924, (2006).
12. J. Ma, W. Liu, and A. Hunter, 'Incomplete statistical information fusion and its application to clinical trials data', in *Proc. of SUM'07*, pp. 89–103, (2007).
13. J. Ma, W. Liu, and A. Hunter, 'The non-archimedean polynomials and merging of stratified knowledge bases', in *Procs. of ECSQARU*, pp. 408–420, (2009).
14. J. Ma, W. Liu, and A. Hunter, 'Inducing probability distributions from knowledge bases with (in)dependence relations', in *Procs. of AAAI*, pp. 339–344, (2010).
15. J. Ma, W. Liu, and P. Miller, 'Event modelling and reasoning with uncertain information for distributed sensor networks', in *Procs. of SUM*, pp. 236–249, Toulouse, France, (2010). Springer.
16. J. Ma, W. Liu, P. Miller, and W. Yan, 'Event composition with imperfect information for bus surveillance', in *Procs. of AVSS*, pp. 382–387. IEEE Press, (2009).
17. P. Miller, W. Liu, F. Fowler, H. Zhou, J. Shen, J. Ma, J. Zhang, W. Yan, K. McLaughlin, and S. Sezer, 'Intelligent sensor information system for public transport: To safely go ...', in *Procs. of AVSS*, pp. 533–538, (2010).
18. K. Mu, Z. Jin, R. Lu, and W. Liu, 'Measuring inconsistency in requirements specifications', in *Procs. of ECSQARU*, volume 3571, pp. 440–451. LNCS, Springer-Verlag, (2005).
19. K. Mu, W. Liu, and Z. Jin, 'A blame-based approach to generating proposals for handling inconsistency in software requirements', *International Journal of Knowledge and Systems Science*, **3(1)**, 1–16, (2012).

20. P. Rossini, ‘Using expert systems and artificial intelligence for real estate forecasting’, in *Procs. of Sixth Annual Pacific-Rim Real Estate Society Conference*, pp. 1–10, Sydney, Australia, (24-27 January 2000).
21. G. Shafer, *A Mathematical Theory of Evidence*, Princeton University Press, 1976.
22. P. Smets, ‘Belief functions’, in *Non-Standard Logics for Automated Reasoning*, eds., P. Smets, A. Mamdani, D. Dubois, and H. Prade, pp. 253–286, (1988).