A Comparison of Merging Operators in Possibilistic Logic

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Abstract. In this paper, we compare merging operators in possibilistic logic. We first propose an approach to evaluating the *discriminating power* of a merging operator. After that, we analyze the computational complexity of existing possibilistic merging operators. Finally, we consider the compatibility of possibilistic merging operators with propositional merging operators.

1 Introduction

Fusion of information coming from different sources is crucial to build intelligent systems. In classical logic, this problem is often called belief merging, which defines the beliefs (resp. goals) of a group of agents from their individual beliefs (resp. goals). It is well-known that priorities or orderings (either implicit or explicit) play an important role in belief merging [18,20,22]. The handling of priorities has been shown to be completely in agreement with possibilistic logic [13]. Possibilistic logic is a weighted logic which attaches to each first-order logic formula a weight belonging to a totally ordered scale, such as $(0, 1]$. An ordering between two formulas is then obtained by comparing the weights attached to them. Possibilistic logic is also known to be a good logical framework for reasoning under inconsistency and uncertainty when only partial information is available.

Many approaches have been proposed for merging uncertain information in possibilistic logic. In the framework of possibilistic logic, each source of uncertain information is represented as a *possibilistic knowledge base*, which is a set of weighted formulas. A possibilistic knowledge base has a unique possibility distribution associated with it. In [3,4], some semantic merging operators were proposed to aggregate *possibility distributions* of original possibilistic knowledge bases, the result is a new possibility distribution. Then the syntactical counterpart of a semantic merging operator is applied to the possibilistic bases, and the result of merging is a possibilistic knowledge base whose possibility distribution is the one obtained by the semantic merging operator. There are two important classes of merging operators, one class contains t-norm operators (for example, the minimum operator) and the other consists of t-conorm operators (for example, the maximum operator). Several adaptive merging rules have been proposed to integrate both the conjunctive and disjunctive operators (e.g., [8,9,12]).

In practice, an important problem is the choice of an appropriate merging approach. To facilitate the choice among different merging approaches, we need some criteria to evaluate the merits and drawbacks of each approach.

In this paper, we propose three criteria to evaluate merging operators in possibilistic logic. The first criterion is to evaluate the discriminating power of a merging operator, which is measured by the *nonspecificity function*. After that, we analyze the computational complexity of existing possibilistic merging operators. Finally, we consider the compatibility of possibilistic merging operators with propositional merging operators.

This paper is organized as follows. Section 2 introduces some basic notions of possibilistic logic. We then give a brief overview of merging operators in possibilistic logic in Section 3. In Section 4, we give three criteria to evaluate merging operators in possibilistic logic. Finally, we conclude the paper in Section 5.

2 Preliminaries

In this paper, we consider a propositional language $\mathcal{L}_{PS}$ from a finite set PS of propositional symbols. The classical consequence relation is denoted as $\vdash$. An interpretation is a total function from PS to $\{true, false\}$. Ω is the set of all possible interpretations. An interpretation ω is a model of a formula ϕ, denoted $\omega \models \phi$, iff $w(\phi) = true$. A *classical knowledge base* B is a finite set of propositional formulas. B is consistent iff there exists an interpretation ω such that $\omega(\phi) = true$ for all $\phi \in B$.

Possibilistic Logic: Possibilistic logic [13] is a weighted logic where each classical logic formula is associated with a number in $(0, 1]$. A possibilistic knowledge base (or PKB for short) is the set of possibilistic formulas of the form $B = \{(\phi_i,\ a_i) : i = 1, ..., n\}$. *Possibilistic formula* $(\phi_i,\ a_i)$ means that the necessity degree of ϕ_i is at least equal to a_i. A classical knowledge base $B = \{\phi_i : i = 1, ..., n\}$ corresponds to a PKB $B' = \{(\phi_i, 1) : i = 1, ..., n\}$. A *possibilistic knowledge profile* $\mathcal{E}$ is a multi-set of PKBs. In this paper, we consider only PKBs where every formula ϕ is a classical propositional formula. The classical base associated with B is denoted as B^*, namely $B^* = \{\phi_i | (\phi_i,\ a_i) \in B\}$. A PKB is consistent iff its classical base is consistent.

The semantics of possibilistic logic is based on the notion of a *possibility distribution* π: $\Omega \rightarrow [0, 1]$. $\pi(\omega)$ represents the possibility degree of interpretation ω with available beliefs. A possibility distribution π is *normal* iff there exists $\omega \in \Omega$ such that $\pi(\omega) = 1$. Given a PKB B, a unique *possibility distribution*, denoted by π_B, can be obtained by the principle of minimum specificity [13]. For all $\omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall(\phi_i,\ a_i) \in B, \omega \models \phi_i, \\ 1 - max\{a_i | \omega \not\models \phi_i, (\phi_i, a_i) \in B\} & \text{otherwise.} \end{cases} \quad (1)$$

It has been shown that a PKB B is consistent iff π_B is normal.

Given a PKB B and $a \in (0, 1]$, the a-cut of B is $B_{\geq a} = \{\phi \in B^* | (\phi, b) \in B \ and \ b \geq a\}$. The *inconsistency degree* of B, denoted $Inc(B)$, is defined as $Inc(B) = max\{a_i : B_{\geq a_i}\ is\ inconsistent\}$.

Definition 1. *Let B be a PKB. A formula ϕ is said to be a* possibilistic consequence *of B to degree a, denoted by $B\vdash_{\pi}(\phi, a)$, iff the following conditions hold: (1) $B_{\geq a}$ is consistent; (2) $B_{\geq a}\vdash\phi$; (3) $\forall b>a$, $B_{\geq b}\nvdash\phi$.*

3 Merging Approaches in Possibilistic Logic

According to [1], there are two different categories of approaches for merging PKBs. The first category of approaches resolves inconsistency after merging and result in a unique consistent base (e.g. [3, 4, 6, 25, 26]. By contrast, the second category of approaches tolerates inconsistency and cope with them [1, 5, 24]. Because of page limit, we consider only some important merging approaches belonging to the first category.

Given n PKBs B_1,...,B_n, a semantic combination operator $\oplus$ maps possibility distributions π_1,...,π_n into a new possibility distribution. The semantic combination can be performed easily when $\oplus$ is associative. That is, we have $\pi_{\oplus}(\omega) = (...((\pi_1(\omega) \oplus \pi_2(\omega)) \oplus \pi_3(\omega)) \oplus ...) \oplus \pi_n(\omega)$. When the operator is not associative, it needs to be generalized as an unary operator defined on a vector $(\pi_1, ..., \pi_n)$ of possibility distributions such that:

1. $\oplus(1, ..., 1) = 1$, and
2. if $\forall i = 1, ..., n,\ \pi_i(\omega)\geq\pi_i(\omega')$ then $\oplus(\pi_1(\omega), ..., \pi_n(\omega))\geq\oplus(\pi_1(\omega'), ..., \pi_n(\omega'))$.

Two classes of aggregation operator which are commonly used are t-norm (denoted as tn) and t-conorm (denoted as ct). Examples of t-norms are the minimum operator and the product operator and examples of t-conorms are the maximum operator and the "probabilistic sum" operator defined by $a\oplus b = a + b - ab$.

The merged possibility distribution of a t-norm operator may be not normal. In that case, we may think of renormalizing π_{tn}. Let π be a possibility distribution which is not normal, π_N be the possibility distribution renormalized from π. Then π_N should satisfy the following conditions:

1. $\exists\omega$, $\pi_N(\omega) = 1$,
2. if π is normal then $\pi_N = \pi$,
3. $\forall\omega, \omega'$, $\pi(\omega) < \pi(\omega')$ if and only if $\pi_N(\omega) < \pi_N(\omega')$.

For example, let $h(\pi_{tn}) = max_{\omega\in\Omega}\{\pi_{tn}(\omega)\}$, the following equation provides a normalization rule.

$$\pi_{N,tn}(\omega) = \begin{cases} 1 & \text{if } \pi_{tn}(\omega) = h(\pi_{tn}), \\ \pi_{tn}(\omega) & \text{otherwise.} \end{cases} \tag{2}$$

The normalization rule defined by Equation 2 resolves inconsistency because the inconsistency degree of any PKB associated with $\pi_{N,tn}$ is zero. Other normalization rules can be found in [3].

The syntactic generalization for a semantic operator can be carried out as follows.

Proposition 1. *[4] Let $\mathcal{E} = \{\mathcal{B}_1, ..., \mathcal{B}_n\}$ be a set of n PKBs and $(\pi_1, ..., \pi_n)$ be their associated possibility distributions. Let $\pi_{\mathcal{B}_\oplus}$ be the result of combining $(\pi_1, ..., \pi_n)$ with $\oplus$. The PKB associated with $\pi_{\mathcal{B}_\oplus}$ is:*

$$\mathcal{B}_\oplus = \{(D_j, 1 - \oplus(x_1, ..., x_n)) : j = 1, ..., n\}, \tag{3}$$

where D_j are disjunctions of size j between formulas taken from different $\mathcal{B}_i$'s ($i = 1, ..., n$) and x_i is equal to $1 - a_i$ if ϕ_i belongs to D_j and 1 if it does not.

By Equation 3, the PKBs, which are the syntactical counterparts of semantic merging using a t-norm tn and a t-conorm ct are the following knowledge bases respectively [3]:

$$\begin{aligned}\mathcal{B}_{tn} = B_1 \cup B_2 \cup \{(\phi_i \vee \psi_j, ct(a_i, b_j)) | (\phi_i, a_i) \in B_1 \\ and\ (\psi_j, b_j) \in B_2\},\end{aligned} \tag{4}$$

$$\mathcal{B}_{ct} = \{(\phi_i \vee \psi_j, tn(a_i, b_j)) | (\phi_i, a_i) \in B_1,\ (\psi_j, b_j) \in B_2\}. \tag{5}$$

By Equation 4, the PKB $\mathcal{B}_{tn}$ may be inconsistent. Let $\pi_{N,tn}$ be the possibility distribution obtained by Equation 2, then the PKB associated with it has the following form:

$$\mathcal{B}_{N,tn} = \{(\phi_i, a_i) : (\phi_i, a_i) \in \mathcal{B}_{tn}\ and\ a_i > Inc(\mathcal{B}_{tn})\}. \tag{6}$$

$\mathcal{B}_{N,tn}$ restores consistency of $\mathcal{B}_{tn}$ by dropping formulas whose weights are less than or equal to the inconsistency degree of $\mathcal{B}_{tn}$. We call the merging operator obtained by Equation 6 a normalized conjunctive merging operator. It is clear that $\mathcal{B}_{N,tn}$ may drop too much information from $\mathcal{B}_{tn}$ if $Inc(\mathcal{B}_{tn})$ is large, for example, 0.8. This is because possibilistic inference suffers from the *drowning problem* [2].

Example 1. Let $B_1 = \{(p, 0.9), (q, 0.7)\}$ and $B_2 = \{(\neg p, 0.8), (r, 0.6), (p \vee q, 0.5)\}$. Suppose the operator is the maximum, then by Equation 5, we have $\mathcal{B}_{max} = \{(p \vee r, 0.6), (p \vee q, 0.5), (\neg p \vee q, 0.7), (q \vee r, 0.6)\}$. It is clear that the maximum based merging operator is very *cautious*, that is, all the formulas are weakened as disjunctions. By constrast, if we choose the minimum, then by Equation 4, we have $\mathcal{B}_{min} = \{(p, 0.9), (\neg p, 0.8), (q, 0.7), (r, 0.6), (p \vee q, 0.5)\}$. $\mathcal{B}_{min}$ is inconsistent. Suppose we apply the normalization rule (Equation 2) to the possibility distribution associated with $\mathcal{B}_{min}$, then by Equation 6 the PKB associated with the normalized possibility distribution is $\mathcal{B}_{N,min} = \{(p, 0.9)\}$. $(q, 0.7)$ and $(r, 0.6)$ are not involved in conflict between B_1 and B_2, but they are deleted after merging.

Example 1 illustrates that the merging methods based on t-conorms are too cautious when most of the formulas are not involved in conflict while the renormalization based merging method may delete too much original information from the resulting knowledge base. According to [7], when knowledge bases are consistent with each other, it is preferable to use a t-norm based merging method. The maximum based merging method is preferable to the minimum based merging method (or any other t-norm based merging method) only if the inconsistency degree of $B_1 \cup ... \cup B_n$ is 1; that is, if there is a strong conflict among the sources of information.

3.1 Adaptive Combination Rules

Because of the pros and cons of the t-norm and t-conorm operators, it is not advisable to use only one of them when information sources partially agree with each other and only some of them are reliable. Several adaptive merging rules have been proposed to integrate both the t-norm and t-conorm operators.

Conflict-respectful combination rule: the normalization rules resolve the conflict between sources. However, as pointed out in [8], they may be very sensitive to rather small variations of possibility degrees around 0. In other word, the rules are not continuous in the vicinity of the total conflict expressed by $h(\pi_{\oplus}) = 0$. An adaptive combination rule was proposed in [Dubois and Prade 1994] to discount the result given by normalization rule by the inconsistency degree of the conjunctively merged knowledge bases, i.e. $1-h(\pi_{B_{\oplus}})$. That is, we have the following modified conjunctive combination rule: $\forall\omega$,

(CoR-N) $\pi_{\oplus,CoR-N}(\omega) = max(\pi_{N,\oplus}(\omega), 1-h(\pi_{\oplus}))$.

An adaptive rule in [8] was proposed which considered j sources out of all the sources, where it was assumed that these j sources are reliable. Since it was not known which j sources were reliable, all the subsets with cardinality j were considered. The intermediary conjunctively merged results are then merged disjunctively. Given a possibilistic profile $\mathcal{E} = \{B_1, ..., B_N\}$ with π_i being the possibility distribution of B_i, the adaptive rule is defined as

$$\pi_{(j)}(\omega) = max_{J\subseteq\mathbf{N},|J|=j}\{min_{i\in J}\{\pi_i(\omega)|\omega\in\Omega\}\}, \tag{7}$$

where $\mathbf{N} = \{1, ..., N\}$.

A method to decide the value of j was given in [12]: let

$$m = max\{|T| : h(T) = 1\}, \tag{8}$$

$$n = max\{|T| : h(T) > 0\}, \tag{9}$$

where $T\subseteq\mathcal{E}$ and $h(T) = max_{\omega}min_{B_i\in T}\pi_i(\omega)$, then, j is defined as m and N is defined as n, where n indicates that these n sources at least partically consistent and among them j sources are completely consistent.

Another adaptive rule in [11]: the rule proposed in [11] utilizes the maximum and the minimum operators. This operator is extended to more than two sources in [12] based on the adaptive rule in Equation 7. It is defined as follows:

$\pi_{AD}(\omega) = max(\frac{\pi_{(n)}(\omega)}{h(n)}, min(\pi_{(m)}(\omega), 1-h(n)))$,

where $h(n) = max\{h(T)||T| = n\}$ as defined previously. This operator utilizes both $\pi_{(n)}$ and $\pi_{(m)}$. It first *renormalizes* $\pi_{(n)}$ which is the result of conjunctive merge of n sources and then applies the maximum operator to the conjunctively merged results of all subgroups with cadinality m, before combines the two parts using the maximum operator. So it is more adaptive and context dependent than adaptive rule in Equation 7.

MCS-based adaptive merging in [10]: an adaptive operator based on maximal consistent subsets (MCS) of $\mathcal{E}$ was proposed in [10]. Suppose $\mathcal{E}_1, ..., \mathcal{E}_k$

are all the maximal consistent subsets of $\mathcal{E}$, then the MCS-based operator is defined as

$\pi_{MCS}(\omega) = max_{i=1,...,k} min_{B_j \in \mathcal{E}_i} \pi_{B_j}(\omega)$

Split-Combination merging approach in [26]: The general idea of the Split-Combination (S-C) approach can be described as follows. Given a set of PKBs B_i, where $i = 1, ..., n$, in the first step, we split them into $B_i = \langle C_i, D_i \rangle$ with regard to a splitting method. In the second step, we combine all C_i by a t-conorm operator (the result is a PKB C) and combine all D_i by a t-norm operator (the result is a PKB D). The final result of the S-C combination method, denoted by B_{S-C}, is $C \cup D$. Different S-C methods can be developed by incorporating different ways of splitting the knowledge bases, while retaining the general S-C approach. Two different splitting methods have been given. One is called the Incremental splitting (I-S) method and the other is called the *free-formula based* splitting (F-S) method. The I-S method first searching for the splitting value using an incremental algorithm and then splits each PKB B_i into two subbases using the splitting value. The F-S method splits a PKB B into two subbases such that one of them contains formulae which are not in conflict in B and the other contains formulae which are in conflict. We call the merging operator based on the *I-S* method as Incremental Split-Combination (*I-S-C*) merging operator and the merging operator based on the free-formula based method as free-formula based split-combination (*F-S-C*) merging operator. Note that the *F-S-C* merging operator does not have a semantic counterpart.

In next section, we propose three evaluation criteria and compare existing merging operators in possibilistic logic with respect to them. As for merging operators in classical logic, we use Δ_X to denote the merging operator X. For example, Δ_{ct} is the t-conorm based merging operator.

4 Evaluation Criteria

4.1 Discriminating Power

Information is generally hard to obtain. Therefore, a good merging operator should preserve as much original information as possible. Inferential power is an important factor for evaluating a merging operator. Given two merging operators, it is natural to prefer the one leading to a merged base which can non-trivially infer more information. However, most merging operators can not be compared with regard to it. Therefore, we propose another evaluation method which is based on the *measure of non-specificity*.

In [16], a measure of possibilistic uncertainty, called *nonspecificity*, was proposed to generalize the Hartley measure of information [15]. Given a possibility distribution π on $\Omega = \{\omega_1, ...\omega_n\}$, $\pi(\omega_i)$ $(i = 1, ..., n)$ are reordered as $\pi_1 = l \geq \pi_2 \geq ... \geq \pi_n$, where l may be less than 1 (in that case, π is not normal). Let $\pi_{i+1} = 0$. The measure of nonspecificity of π is

$$H(\pi) = \frac{1}{l} \Sigma_{j=1}^{n} (\pi_j - \pi_{j+1}) log_2 j \tag{10}$$

Given two PKBs B_1 and B_2, we say the quality of B_1 is better than that of B_2 if $H(\pi_{B_1}) < H(\pi_{B_2})$, where π_{B_i} $(i = 1, 2)$ are possibility distributions of B_i.

Nonspecificity measures information carried by a possibility distribution. Now an interesting question is whether a knowledge base with stronger inferential power also has better quality. The answer is positive according to the following proposition.

Proposition 2. [1] *Given two consistent PKBs B_1 and B_2, π_{B_1} and π_{B_2} are possibility distributions associated with them. If $B_1 \vdash_\pi (\phi, a)$ for any $(\phi, a) \in B_2$, then $H(\pi_{B_1}) \leq H(\pi_{B_2})$.*

Proposition 2 agrees with intuition that stronger inferential power means more information. Therefore, nonspecificity measures the discriminating power of a merging operator.

In the following, we define an ordering to compare two possibilistic merging operators with respect to their discriminating power.

Definition 2. *Let Δ_1 and Δ_2 be two merging operators in possibilistic logic. An ordering between Δ_1 and Δ_2, denoted $\preceq_{DP}$, is defined as:*

$$\Delta_1 \preceq_{DP} \Delta_2 \quad iff \quad \forall \mathcal{E},\ H(\pi_{\Delta_1(\mathcal{E})}) \leq H(\pi_{\Delta_2(\mathcal{E})}),$$

where $\pi_{\Delta_i(\mathcal{E})}$ $(i = 1, 2)$ are the possibility distributions associated with $\Delta_i(\mathcal{E})$. It is clear that $\preceq_{DP}$ is a partial preorder, i.e. it is reflective and transitive. As usual, we define the strict order $\Delta_1 \prec_{DP} \Delta_2$ iff $\Delta_1 \preceq_{DP} \Delta_2$ and $\Delta_2 \not\preceq_{DP} \Delta_1$. If $\Delta_1 \prec_{DP} \Delta_2$, then we say that Δ_1 is stronger than Δ_2 *w.r.t* their discriminating power.

We compare existing merging operators in possibilistic logic *w.r.t* their discriminating power.

Proposition 3. *When $\mathcal{E}$ is consistent, then $\Delta_{N,tn} \prec_{DP} \Delta_{ct}$. However, this relationship does not hold when $\mathcal{E}$ is inconsistent.*

Proposition 4. *The normalized conjunctive merging operator has stronger discriminating power than the conflict-respectful merging operator, i.e., $\Delta_{N,tn} \prec_{DP} \Delta_{CoR-N}$.*

Proposition 5. *Both the j-source and MCS based adaptive merging operators have stronger discriminating power than the t-conorm based one. That is, $\Delta_{\{j\}} \prec_{DP} \Delta_{ct}$ and $\Delta_{MCS} \prec_{DP} \Delta_{ct}$.*

Proposition 6 (26). *Let $\mathcal{E} = \{B_1, ..., B_n\}$ be a set of n PKBs. Let Δ_{I-S-C}, Δ_{F-S-C} and Δ_{ct} be the I-S-C operator, F-S-C operator and t-conorm based merging operator respectively. We have $\Delta_{I-S-C} \prec_{DP} \Delta_{ct}$ and $\Delta_{F-S-C} \prec_{DP} \Delta_{ct}$.*

It has been shown in [26] that *I-S-C* merging operator and normalized conjunctive merging operator are not comparable *w.r.t* the discriminating power. However, we have the following proposition.

[1] All proofs of propositions can be found in a PhD thesis which is available at http://gqi.limewebs.com/thesis.pdf.

Proposition 7. *[26] Let B_1 and B_2 be two PKBs. Suppose the t-norm tn and t-conorm ct which are used to define the I-S-C operator is the minimum and an arbitrary t-conorm respectively. Let γ be the splitting point for the I-S-C operator which is obtained by the splitting algorithm. Suppose $\gamma = Inc(B_1 \cup B_2)$, then $B_{min,N2} \subseteq B_{I-S-C}$, but not vice versa.*

4.2 Computational Complexity

Computational complexity has been adopted as an important criterion to evaluate a solution in many AI problems, such as belief revision [14]. From an application point of view, computational efficiency is an important requirement when one selects a merging operator. It has been shown in [21] that in the propositional setting, merging is generally a hard task in the worst case. It is clear that computationally more efficient operators are preferable to more complex ones.

We assume that the reader is familiar with computational complexity (more details can be found in [17]) and we consider the following classes located at the first or the second level of polynomial hierarchy.

- $\Delta_2^p = P^{NP}$ is the class of decision problems solvable in polynomial time by a deterministic Turing machine equipped with an NP oracle.
- Σ_2^p is the class of decision problems solvable in polynomial time by a non-deterministic Turing machine equipped with an NP oracle.
- Π_2^p=co-Σ_2^p, where co-Σ_2^p is the class of problems whose answer is always the complement of those in Σ_2^p.

Note that the following relations have been conjectured in complexity theory:
P⊂NP, P⊂co-NP, NP≠co-NP, NP⊂Δ_2^p, co-NP⊂Δ_2^p, co-NP⊂Π_2^p, Δ_2^p=P^{NP}⊂ Σ_2^p

In this paper, we consider *function problems*, problems that require an answer more elaborate than "yes" or "no". Suppose X is a class of decision problems, the function problem associated with X is denoted by FX. For example, $F\Delta_2^p$ is the set of problems solvable in polynomial time by a machine with access to an oracle for an NP problem.

In the following, we analyze the computational complexity of existing merging operators.

Proposition 8. *Generating a consistent PKB by a t-conorm based merging operator is in FP and generating a consistent PKB by a normalized conjunctive merging operator is in $F\Delta_2^p$.*

By Proposition 8, the t-conorm based merging operator is tractable. However, since it is a cautious operator that may drop too much information, we do not advocate using it unless the sources are strongly in conflict. Therefore, before applying the t-conorm based merging operator, we should at least check the consistency of the union of original sources, which is a NP-complete task.

The computational complexity of the conflict-respectful operator is not harder than that of the normalized conjunctive merging operator.

Proposition 9. *Generating a consistent PKB by a conflict-respectful merging operator is in* $F\Delta_2^p$.

Proposition 10. *Suppose* $j = m$*, where* m *is defined by Equation (8), then generating a consistent PKB by the* j*-source based merging operator is in* $F\Delta_2^p$. *Generating a consistent PKB by the adaptive merging operator* Δ_{AD} *is in* $F\Delta_2^p$.

The MCS-based merging operator is computationally harder than other operators because it needs to compute all the maximal consistent subbases.

Proposition 11. *Generating a consistent PKB by the MCS-based adaptive merging operator is* $F\Pi_2^p$*-hard.*

We now consider the computational complexity results given in [26]

Proposition 12. *Generating a consistent PKB using the* I*-*S*-*C *operator and* F*-*S*-*C *operator are in* $F\Delta_2^p$ $(\mathcal{O}(n))$ *and* $F\Pi_2^p$ *respectively.*

According to above propositions, the t-conorm based merging operator is computationally easier than other merging operators. The computational complexities of most of merging operators in possibilistic logic remain at the first level of polynomial hierarchy, a low level of polynomial hierarchy. Finally, the MCS-based merging operator and *F-S-C* merging operator are computationally harder than other operators, i.e. their computational complexities are located at leat at the second level of polynomial hierarchy.

4.3 Compatibility with Propositional Merging Operators

It has been pointed out in [13] that when the necessity degrees of all the possibilistic formulas are taken as 1, possibilistic logic is reduced to classical logic. So classical logic is a special case of possibilistic logic in which all the formulas have the same level of priority. Therefore, merging methods in possibilistic logic can be directly applied to merge classical knowledge bases. When comparing two different operators which are not comparable with regard to other criteria, we prefer possibilistic merging operators which are well-behaved in classical logic to those which are not. In classical logic, the main criterion for comparing merging operator is the rationality properties. In this section, we analyze the logical properties of different possibilistic merging operators in the flat case.

We first introduce the postulates for characterizing a propositional merging operator proposed in [18].

Definition 3. *Let* Δ *be a propositional operator which assigns to a set of knowledge bases* E *a knowledge base* $\Delta(E)$*. Let* E_1 *and* E_2 *be two sets of knowledge bases,* K *and* K' *be two knowledge bases.* Δ *is a propositional merging operator iff it satisfies the following postulates:*

(A1) $\Delta(E)$ *is consistent.*
(A2) If E *is consistent, then* $\Delta(E) \equiv \bigwedge E$*, where* $\bigwedge E = \wedge_{K_i \in E} K_i$.

(A3) If $E_1 \equiv E_2$, then $\Delta(E_1) \equiv \Delta(E_2)$.
(A4) If $K \wedge K'$ is not consistent, then $\Delta(\{K\} \sqcup \{K'\}) \not\models K$.
(A5) $\Delta(E_1) \wedge \Delta(E_2) \models \Delta(E_1 \sqcup E_2)$.
(A6) If $\Delta(E_1) \wedge \Delta(E_2)$ is consistent, then $\Delta(E_1 \sqcup E_2) \models \Delta(E_1) \wedge \Delta(E_2)$.

Proposition 13. *Let $E = \{K_1, ..., K_n\}$ be a set of knowledge bases. Let $\Delta_{ct}(E)$ be the resulting knowledge base of a t-conorm based operator. Then $\Delta_{ct}(E) \equiv \{\vee_{i=1}^{n} \phi_i : \phi_i \in K_i, i = 1, ..., n\}$. Let $\Delta_{N,tn}(E)$ be a normalized t-norm operator. Then $\Delta_{N,tn}(E) \equiv \top$.*

According to Proposition 13, the normalized conjunctive operators cannot be directly applied to merging classical knowledge bases because all the information is deleted after merging, whilst the t-conorm based operators take the disjunction of all the knowledge bases as the result of merging.

The logical properties of the operator Δ_{ct} in the flat case are analyzed as follows.

Proposition 14. *The t-conorm based operator Δ_{ct} satisfies (A1), (A3), (A4) and (A5). It does not satisfy other postulates in general.*

Next, we consider the adaptive merging operators.

First, the conflict-respectful merging operators are based on the normalized conjunctive operators. So, they cannot be directly applied to merge classical knowledge base.

The j-source based operator takes the disjunction of all the knowledge bases which are conjunction of j original knowledge bases as the result of merging, that is, we have $\Delta_{\{j\}}(E) = \bigvee_{J \subseteq \mathbf{N}, |J|=j} \{\wedge_{i \in J} K_i\}$.

Proposition 15. *When $j = m$, where m is defined by Equation (8), then the j-source based operator $\Delta_{\{j\}}$ satisfies (A1), (A2), (A3), (A4). It does not satisfies (A5) and (A6) in general.*

According to Proposition 15, when $j = m$, the j-source based operator has good logical properties in the flat case.

The adaptive operator Δ_{AD} is based on the j-source based operator. In the flat case, it is equivalent to the m-source based operator according to the following proposition:

Proposition 16. *The merged result of the adaptive operator Δ_{AD} is equivalent to $\Delta_{\{m\}}(E)$, i.e., $\Delta_{AD}(E) \equiv \Delta_{\{m\}}(E)$.*

The merged result of the MCS-based adaptive operator is the disjunctive of those knowledge bases which are conjunctions of maximal consistent subsets of E. That is, let MAXCONS(E) be the set of maximal consistent subsets of E, we have

$$\Delta_{MCS}(E) = \bigvee_{E_i \in MAXCONS(E)} \bigwedge_{K_{ij} \in E_i} K_{ij}.$$

The operator $\Delta_{MCS}(E)$ is a commonly used syntax-based merging operator in the propositional setting [19].

Proposition 17. *The MCS-based adaptive operator Δ_{MCS} satisfies (A1), (A2), (A3), (A4), (A5). It does not satisfy (A6) in general.*

In [26], tt has been shown that, in the flat case, the *I*-*S*-*C* operator is reduced to the t-conorm based operator. We also have the following results for *F*-*S*-*C* operator [26].

Proposition 18. *The F-S-C merging operator Δ_{F-S-C} satisfies (A1), (A2), (A4) and (A5). However, it does not satisfy* (A3) *and* (A6) *in general.*

According to the above propositions, the operators $\Delta_{N,tn}$ and Δ_{CoR-N} cannot be applied to merging classical knowledge bases. The operator Δ_{AD} and the operator $\Delta_{\{m\}}$ are equivalent in the flat case. The MCS-based adaptive operator satisfies more postulates than all the other operators in possibilistic logic in the flat case.

5 Conclusions

In this paper, we first gave a belief survey of two kinds of merging operators in possibilistic logic: semantic merging operators and adaptive merging operators. We then proposed several evaluation criteria to compare these merging operators. The comparison results will be useful for users to select appropriate merging operator(s) for specific applications.

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