Modeling and reasoning with qualitative comparative clinical Knowledge

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Abstract The number of clinical trials reports is increasing rapidly due to a large number of clinical trials being conducted, it therefore raises an urgent need to utilize the clinical knowledge contained in the clinical trials reports. In this paper, we focus on the qualitative knowledge instead of quantitative knowledge. More precisely, we aim to model and reason with the qualitative comparison (QC for short) relations which consider qualitatively how strongly one drug/therapy is preferred to another in a clinical point of view. To this end, first, we formalize the QC relations, introduce the notions of QC language, QC base, and QC profile; second, we propose a set of induction rules for the QC relations, and provide grading interpretations for the QC bases and show how to determine whether a QC base is consistent. Furthermore, when a QC base is inconsistent, we analyze how to measure inconsistencies among QC bases, and we propose different approaches to merging multiple QC bases. Finally, a case study on lowering intraocular pressure is conducted to illustrate our approaches.

Keywords Qualitative Comparison Relation; Clinical Knowledge; Biomedical Knowledge; Grading Interpretation; Inconsistency Measure; Pair-wise merging; Prioritized merging; Induction Rule

1 Introduction

There is a huge and rapidly expanding amount of information available for scientists in various online resources. However, this wealth of information has created challenges for scientists who wish to analyze these pieces of knowledge. One of the key problems that exist is that the knowledge may be uncertain, incomplete and inconsistent. Scientists therefore need tools that are tolerant of uncertainty, incompleteness and inconsistency in order to query and merge scientific knowledge.

To illustrate, consider the area of clinical trials. A clinical trial is a study that compares the effect of one medication (or other treatment) against another [13]. A huge number of clinical trials have been carried out in the last few decades and new trials are being designed and implemented from time to time. Trial results are a summary of the underlying statistical analysis, usually there is a qualitative statement comparing two drugs of interest. For example, in [1, 5, 6, 7, 12, 14], comparisons of different drugs on the efficacy of lowering intraocular pressure (called IOP reduction) are experimented. The comparisons of drugs on IOP reduction are often expressed in the form of e.g. *travoprost was superior to latanoprost* [6] or *travoprost appears to be equivalent to bimatoprost and latanoprost* [14] or *IOP reduction obtained with travoprost was significantly higher than that obtained with latanoprost* [1] (note travoprost, bimatoprost, latanopros, and timolol are all names of drugs for IOP reduction).

Such comparison sentences can be briefly summarized as the following three relations $p > q, p \simeq q$, and $p \gg q$, respectively. By looking through scientific papers about drugs on IOP reductions and papers on drugs for breast cancer treatments, it shows that such three relations are sufficient and necessary to express qualitative comparison knowledge occurring in clinical trials papers. Hence important questions arising from this observation are (a) how to model and reason with these three types of knowledge; (b) what properties such a model shall posses; (c) how to merge different sets of bases containing such knowledge; and (d) how to measure consistencies and conflict among multiple sets of such knowledge.

Qualitative reasoning has studied in many papers, from the study on signs of quantities [16, 10] to the study on order of magnitude reasoning [24, 22, 8, 11, 2, 25] where basically it presents three relations, i.e., "close to" (Vo), "comparable to" (Co), "negligible w.r.t" (Ne). It seems that our three QC relations are similar to the order of magnitude relations (i.e., Vo vs. \simeq , Co vs. >, and Ne vs. \gg), however, unfortunately the clinical context we study prevents us from using the results of order of magnitude reasoning for drug comparisons. First, the relations introduced in order of magnitude reasoning can not catch > and \gg exactly, e.g., in our context, obviously p > q and q > p are totally different and need to be distinguished, but in order of magnitude reasoning, the Co relation is symmetric (and the "distant from" relation introduced in [8] is also symmetic). What is worse is that the semantic meanings of Co and Ne do not fit for > and \gg , respectively in our context. In fact, in most cases, the effects of different drugs tested in clinical trials are comparable. That is to say, > and \gg are to some extent *sub-relations* of Co while the Ne relation is just of no use in our context. Second, in order of magnitude reasoning rules, addition and concatenation are used on objects like p Vo q implies t.p Vo t.q while in our context, no operation is allowed on objects as each object stands for the effect of a drug, and it is very odd to add or concatenate the effects of two different drugs. According to the above reasons, we will develop a new framework suitable for the clinical trial context. To our knowledge few, if any, papers are on this issue.

We study this topic following the way of studying knowledge bases. Thus, we first propose the QC language as a foundation (just like the propositional language serves as the foundation of knowledge bases). Based on the QC language, we formally define the notions of QC base and QC profile (similar to knowledge bases and knowledge profiles). Moreover, we provide some induction rules for the QC language and

give grading interpretations for the QC relations which are compatible with the induction rules. Based on the grading interpretations, we are also able to define whether a QC base is consistent. Furthermore, we investigate how to measure the inconsistency among multiple QC bases and propose different kinds of merging methods, i.e., the prioritized merging, pair-wise merging and grading interpretation based merging, for different purposes.

The rest of the paper is organized as follows. In Section 2, we define the QC language, notions of QC base and QC profile. In Section 3, we propose some induction rules of QC relations and grading interpretations of QC bases. Next, we propose measures of inconsistency of the QC bases in Section 4. Following this, we propose different merging methods in Section 5. A case study of lowering intraocular pressure is presented in Section 6. Finally, in Section 7, we conclude the paper.

2 Notations and Definitions

To formalize the comparison sentences mentioned in Introduction, in this section, we define the qualitative comparison language $\mathcal{L}_{\mathcal{P}}$ over a finite set \mathcal{P} of symbols (we don't call them atoms because the symbols themselves are not in the language). We will use lower case letters p, q, r, etc to denote the symbols (maybe with subscripts). Three standard connectives, i.e., \gg , >, and \simeq , are used to connect two symbols and thus form sentences in $\mathcal{L}_{\mathcal{P}}$.

Semantically and intuitively, $p \gg q$ means that p is considered significantly more preferred (e.g. more effective, reliable, probable, etc) than q, e.g., *IOP reduction obtained with travoprost was significantly higher than that obtained with latanoprost* [1]; p > q means that p is more preferred (but not significantly) than q, e.g., *travoprost is more effective than timolol in lowering IOP in patients with open-angle glaucoma or ocular hypertension* [14]; and $p \simeq q$ means p is more or less equivalently preferred to q, e.g., *travoprost appears to be equivalent to bimatoprost and latanoprost* [14]. It should be noted that $p \gg q$ does not simply lead to p > q since p > q implies that preference of p to q is not significant.

By abuse of language, we call the relations $p \gg q$, p > q, and $p \simeq q$ propositions in language $\mathcal{L}_{\mathcal{P}}$. So in the following when we mention propositions, we in fact mean these three kinds of sentences if there is no confusion.

A qualitative comparison base (QCB for short) Q is a multi-set of propositions. A qualitative comparison profile (QCP for short) E is a multi-set of QCBs such that $E = \{Q_1, Q_2, \ldots, Q_n\}^1$. For convenience, we denote $\bigcup E$ the union of the QCBs in E, i.e., $\bigcup E = \bigcup_{i=1}^{|E|} Q_i$ where |X| means the cardinality of the set X. $\bigcup E$ can also be seen as a QCB itself.

3 Basic Properties

In this section, we discuss some basic properties of the QC relations. First, we define the induction rules on the QC relations. Second, we propose grading interpretations for

¹We use multi-set instead of set as there might be repeated propositions or QCBs.

QCBs and QCPs. The induction rules and the grading interpretations together form a foundation for inconsistency analysis and merging of QCBs.

3.1 Induction Rules

In this subsection, we establish some induction rules for the QC relations.

Let \bowtie represent one of the three connectives \gg , >, and \simeq , and p, q, r be any symbols in \mathcal{P} . Then, we have the following induction rules:

reflection rule: $p \simeq p$ *symmetry rule:* $p \simeq q$ implies $q \simeq p$ *totality rule:* $p \simeq q$ or q > p or p > q or $p \gg q$ or $q \gg p$. *transition rules:*

1. $p \bowtie_1 q$ and $q \bowtie_2 r$ implies $p \gg r$ if at least one of \bowtie_1 and \bowtie_2 is \gg .

- 2. $p \bowtie q$ and $q \simeq r$ implies $p \bowtie r$.
- 3. $p \simeq q$ and $q \bowtie r$ implies $p \bowtie r$.
- 4. p > q and q > r implies $p \gg r$.

The *reflection* and *symmetry* rules seem intuitive and uncontroversial. The *totality* rule actually shows the range we consider, i.e., we only consider the three relations and any two symbols should satisfy at least one relation. The transition rules, however, al-though reasonable, deserve some explanation. The most arguable transition rule might be the fourth one (the last one) which says p > q and q > r implies $p \gg r$. This rule is an empirical rule. From our investigation, this rule is particularly adequate in the clinical context. When comparing the effects of two drugs, we find that the effects are more or less comparable², e.g., drug A may reduce the IOP 6.8mm Hg, drug B may reduce 4.5mm Hg, etc. In this sense, we consider it rational to combine two discrepancies into a big discrepancy, namely the fourth transition rule. The other three transition rules are consistent with the idea of order of magnitude reasoning and not surprising. Nevertheless, the use of reflection rule and symmetric rule do not propagate uncertainty whilst the use of transition rules may increase the uncertainty. Hence if a proposition $p \bowtie q$ is induced by applying the transition rules, then this proposition is not as reliable as the original propositions.

Proposition 1 Let p, q, r be symbols, then $p \gg r$ and q > r implies $p \gg q$ or p > q.

Proof: Please see appendix for all the proofs.

This proposition shows that the converse of the fourth transition rule does not hold, i.e., \gg is not simply equivalent to two >s.

Definition 1 A QCB Q is called simeq closed iff for any proposition $p \simeq q \in Q$, $q \simeq p$ is also in Q.

²This may because when clinicians design their clinical trials, the tested drugs they choose are already considered effective. The aim of trials is to obtain to what extent the drugs are effective and to compare such drugs.

We need QCBs to be *simeq closed* in order to facilitate the induction process. Moreover, as mentioned above, putting $q \simeq p$ into Q does not bring more uncertainty (and of course does not bring more information).

A QCB Q can be easily extended to be a simeq closed QCB Q_{sc} by the following procedure.

1. Set $Q_{sc} = Q$.

2. For each proposition $p \simeq q \in Q$, let $Q_{sc} = Q_{sc} \cup \{q \simeq p\}$.

Obviously, Q and its corresponding simeq closed QCB Q_{sc} provide the same information.

The necessity of extending a QCB to be simeq closed is illustrated by the following example.

Example 1 Let $Q = \{p \gg q, r \simeq q\}$ be a QCB, then its corresponding simeq closed QCB is $Q_{sc} = \{p \gg q, r \simeq q, q \simeq r\}$. We cannot use the transition rules directly on Q since there are not enough propositions to allow the inductions rules to work. However, on Q_{sc} we can induce $p \gg r$ by the transition induction rules.

We define the *simeq closed* QCPs below.

Definition 2 A QCP E is called simeq closed iff $\forall Q \in E$, Q is simeq closed and for each symbol p appeared in E, $p \simeq p$ is in E.

A QCP E can be easily extended to be a simeq closed QCP E_{sc} by the following procedure.

- 1. Set $E_{sc} = \emptyset$.
- 2. For each QCB $Q \in E$, let $E_{sc} = E_{sc} \cup \{Q_{sc}\}$ where Q_{sc} is the corresponding simeq closed QCB of Q.
- 3. For all symbols p_1, \ldots, p_n appeared in E, we construct a QCB $Q_{sim} = \{p_1 \simeq p_1, \ldots, p_n \simeq p_n\}$ and let $E_{sc} = E_{sc} \cup \{Q_{sim}\}$.

QCP E and its corresponding simeq closed set E_{sc} is equivalently informative. The reason why we should include the propositions like $p \simeq p$ into E_{sc} is briefly demonstrated by the following example.

Example 2 Let $E = \{\{p > q\}, \{q > p\}\}$, then $E_{sc} = \{\{p > q\}, \{q > p\}, \{p \simeq p, q \simeq q\}\}$. By the transition induction rules, we can induce $p \gg p$ and $q \gg q$ in both E and E_{sc} which are apparently counterintuitive. Now let us examine them in detail. In QCP E, since $p \gg p$ and $q \gg q$ are not contradicted with either p > q or q > p, we have no reason to reject such two induced propositions. However, in E_{sc} , the existence of $p \simeq p$ and $q \simeq q$ can be used to exclude the induced propositions $p \gg p$ and $q \gg q$ (because the transition induction rules are fault prone, such two propositions are taken to be less reliable than the original ones). This example will be discussed further after we have introduced the priority level of propositions in Section 4.

In the following, all the mentioned QCBs and QCPs are already extended to the simeq closed version. Moveover, by abuse of notations, we still use Q (not Q_{sc}) to denote the simeq closed QCB and E to denote the simeq closed QCP.

3.2 Grading Interpretations

In this subsection, we introduce grading interpretations for the QCBs and QCPs and show that they match the induction rules proposed above.

Definition 3 Let f be a mapping from \mathcal{P} to a set of ordinal numbers. We call f a grading interpretation of a QCB Q iff it satisfies:

- 1. If $p \simeq q \in Q$, then f(p) f(q) = 0.
- 2. If $p > q \in Q$, then f(p) f(q) = 1.
- 3. If $p \gg q \in Q$, then $f(p) f(q) \ge 2$.

Moreover, f is called a grading interpretation of a QCP E iff f is a grading interpretation of $\bigcup E$.

A grading interpretation f in fact depicts the *grades* obtained by the symbols in the QCB (or QCP). From this perspective, f(p) - f(q) = 0 should be understood as p and q are in the same grade, hence $p \simeq q$; f(p) - f(q) = 1 as p is one grade higher than q hence p > q; and $f(p) - f(q) \ge 2$ as p is at least two grades (and possible more) higher than q hence $p \gg q$. This interpretation also helps to rational the fourth transition rule defined earlier.

With the help of grading interpretations, we now define when a QCB (or a QCP) is called consistent.

Definition 4 A QCB Q is called **consistent** iff there exists a function f such that f is a grading interpretation of Q. A QCP E is called **consistent** iff $\bigcup E$ is consistent.

Thus in a consistent QCB (or a QCP), a grading interpretation can be used to analyze the relations between symbols. As a consequence, the theorem below shows that the grading interpretations perfectly matches the induction rules in a consistent QCB and QCP.

Theorem 1 Let f be a grading interpretation of a consistent QCB Q (resp. QCP E), then it induces the induction rules on Q (resp. E).

This theorem reveals that the induction rules and the grading interpretations are to some extent equivalent. Hence, we can use the grading interpretations to assist the reasoning of induction rules.

Obviously, there can be many grading interpretations for a consistent QCP (or QCB). We want to find a representative grading interpretation for the QCPs (or QCBs).

Definition 5 Let \mathcal{F} be the set of all grading interpretations for a consistent QCP (or QCB). We define a total pre-order relation \leq on \mathcal{F} such that $\forall f, g \in \mathcal{F}$, we write $f \leq g$ iff for all symbols p_1, \ldots, p_n appeared in the QCP (or QCB), $\sum_{i=1}^n f(p_i) \leq \sum_{i=1}^n g(p_i)$.

Definition 6 Let \mathcal{F} be the set of all grading interpretations for a consistent QCP (or QCB), then $\forall f \in \mathcal{F}$, f is called **minimal** iff $f \in min(\mathcal{F}, \leq)$.

We hope that the minimal grading interpretation is unique, thus it can be taken as a good representative of the consistent QCP (or QCB). Fortunately, the following theorem does show that the minimal grading interpretation is unique.

Theorem 2 Let \mathcal{F} be the set of grading interpretations for a consistent QCP (or QCB), then $|min(\mathcal{F}, \leq)| = 1$.

Example 3 Let a QCB Q be $Q = \{p \simeq q, p \gg r, s > r, r > t, p \gg t, q > s, l \gg t, q > t, q > s, l \gg t, q > s, l \gg t, q > t, q > s, l \gg t, q > t, q > s, l \gg t, q > t, q >$ $q, l \gg t$. Then the unique minimal grading interpretation f is: f(l) = 5, f(p) = 5f(q) = 3, f(s) = 2, f(r) = 1, f(t) = 0.Let a QCB Q' be Q' = { $p \simeq q, p \gg r, q > r$ }. Because there are no grading

interpretations available for Q', Q' is inconsistent.

Inconsistency Measures of Qualitative Comparison 4 Bases

In this section, we analyze the inconsistency of multiple QCBs, or, a QCP E. First, we consider the priority levels of propositions in E. Second, based on these priority levels, we investigate how to measure the inconsistency of E.

4.1 **Priority level**

Recall that in the previous section, we have commented that the transition induction rules should be applied with caution because the transition rules are fault prone. If one proposition $p \bowtie_1 q$ appears directly in a QCB $Q \in E$, while another proposition $p \bowtie_2 q$ is obtained by induction from the transition rules, where $\bowtie_1, \bowtie_2 \in \{\gg, >, \simeq\}$, then it is natural that $p \bowtie_1 q$ should be taken as more reliable than $p \bowtie_2 q$. Hence $p \bowtie_1 q$ should have a higher priority than that of $p \bowtie_2 q$ when these two propositions are considered. Moreover, if a proposition $p \bowtie_3 q$ is obtained by using the transition rules at least i times, while $p \bowtie_4 q$ is obtained by using the transition rules at least j times, then $p \bowtie_3 q$ should have a higher priority than $p \bowtie_4 q$ whenever i < j. That is, the more times the transition rules are applied to obtain a proposition, the less reliable the proposition is.

Example 4 (Example 2 Revisited) In Example 2, we have commented that in E_{sc} , the existence of $p \simeq p$ and $q \simeq q$ can be used to exclude the induced propositions $p \gg p$ and $q \gg q$. This is exactly what priority levels can do. As the priorities of $p \simeq p$ and $q \simeq q$ are higher than those of $p \gg p$ and $q \gg q$ (as they are obtained by the transition rules). When we consider the QC relation between p and p (resp., q and q), $p \simeq p$ (resp., $q \simeq q$) will be considered as more reliable. Therefore the existence of $p \simeq p$ and $q \simeq q$ suppresses $p \gg p$ and $q \gg q$.

The above discussion, to some extent, tells that the least number of times that the transition rules are used for inducing a proposition can be taken as the priority level of the proposition. Any existing proposition has the highest priority level 0, while a proposition obtained by using the transition rules at least *i* times has priority level *i*. In Example 4, the proposition $p \simeq p$ has priority level 0, and $p \gg p$ has priority level 1, if we want to obtain $p \gg q$ or $q \gg p$, as the least number of times to use transition rules is 2, the priority levels of $p \gg q$ and $q \gg p$ are 2.

It should be noted that the use of reflection rule and symmetry rule do not affect the priority level of a proposition. This is because they are necessarily true. Intuitively, it holds universally that $p \simeq p$, and if $p \simeq q$ then $q \simeq p$ must be true. Semantically, pand p itself will definitely be in the same grade, and if p is in the same grade as q, then definitely q is in the same grade as p. Thus the reflection and symmetry rule will not lead to any possible error (i.e., any possible upgrading or degrading). The transition rules, however, are fault prone, thus the more times the transition rules are used, the less reliability they are. That is why the least number of times of using the transition rules can be taken as the priority level of a proposition being induced.

We formalize the above discussion as the following.

Definition 7 A transition induction process on a QCB Q_1 constructs a QCB Q_2 as follows.

- 1. Set $Q_2 = \emptyset$.
- 2. For each pair of propositions $p_1 \bowtie_1 q_1$ and $p_2 \bowtie_2 q_2$ in Q_1 , if these two propositions can induce a third proposition $p_3 \bowtie_3 q_3$ following a transition rule and $p_3 \bowtie_3 q_3 \notin Q_1$, then let $Q_2 = Q_2 \cup \{p_3 \bowtie_3 q_3\}$.

We hope that the constructed QCB Q_2 to be simeq closed, then we do not need another extending process. Fortunately it is.

Proposition 2 The QCB Q constructed by Definition 7 is simeq closed.

From a given QCP E (in fact its corresponding QCB $\bigcup E$), if we repeatedly apply the transition induction process, we obtain the following prioritized simeq closed QCBs.

Let Q_0 be $\bigcup E$, and Q_i be the constructed QCB from $(\bigcup_{i=0}^{n-1} Q_i)$ based on Definition 7, $1 \le i \le n$. Let $E(n) = (Q_0, \ldots, Q_n)$ denote the list of QCBs constructed from E up to n times induction of transition rules, then E(n) is called a prioritized QC base, and propositions in Q_i is more important than that in Q_j if i < j. The propositions in E(n) is $\cup_i Q_i$.

Now we give the formal definition of priority level for each proposition.

Definition 8 Let $E(n) = (Q_0, \ldots, Q_n)$ be a prioritized QC base. A proposition $p \bowtie q$ has priority level i iff $p \bowtie q \in Q_i$ and $p \bowtie q \notin \bigcup_{j=0}^{i-1} Q_j$. If $p \bowtie q \notin \bigcup_{j=0}^n Q_j$, then we define the priority level of $p \bowtie q$ as $+\infty$. Here $\bowtie \in \{\gg, >, \simeq\}$.

For convenience, for proposition $p \bowtie q$ with priority level *i*, we simply denote $(p \bowtie q, i) \in E(n)$.

Proposition 3 For any two symbols, two propositions $p \simeq q$ and $q \simeq p$ always have the same priority level with respect to E(n).

Example 5 Let $E = \{\{p > q\}, \{q > p\}, \{p \simeq p, q \simeq q\}\}$, we have $E(n) = (Q_0, Q_1, Q_2, \dots, Q_n)$ such that $Q_0 = \{p > q, q > p, p \simeq p, q \simeq q\}$, $Q_1 = \{p \gg p, q \gg q\}$, $Q_2 = \{p \gg q, q \gg p\}$, and for $i \ge 3$, $Q_i = \emptyset$.

Among these propositions, we have in particular $(p \simeq p, 0), (p \gg p, 1) \in E(n)$ and $(p \gg q, 2) \in E(n)$.

Because the transition rules should be applied with caution, we may only want to consider induced propositions with priority levels no larger than a given threshold t. With the restriction of priority level threshold t, we consider the accessibility relation between two symbols.

Definition 9 Let p and q be two symbols of QCP E. p and q are said to be **absolutely** accessible iff there is a proposition $p \bowtie q$ which has a priority level $i < +\infty$. p and qare said to be relatively accessible iff there is a proposition $p \bowtie q$ which has a priority level i < t where t is the threshold.

Intuitively, p and q are absolutely accessible if a proposition can be established between them after a finite number of steps applying the induction rules. On the other hand, p and q are relatively accessible only if a proposition can be established between them with the number of steps applying the induction rules limited to threshold t.

4.2 Inconsistency measures

In this subsection, we consider how to measure and resolve inconsistency among propositions. We will study two kinds of inconsistencies: one is the inconsistency between propositions like $p \bowtie_1 q$ and $p \bowtie_2 q$, e.g., $p \gg q$ and $p \simeq q$, which suggest some kind of redundancy; the other is the inconsistency between propositions like $p \bowtie_1 q$ and $q \bowtie_2 p$, e.g., p > q and q > p, which express some contradiction or conflict.

For convenience, we first define the difference of two symbols p and q with respect to a proposition.

Definition 10 Let $p \bowtie q$ be a proposition in a QCB Q with $\bowtie \in \{\gg, >, \simeq\}$. The **difference** of p and q in $p \bowtie q$ is defined as:

 $\operatorname{diff}_{\bowtie}(p,q) = \left\{ \begin{array}{ll} 2 & if \bowtie = \gg, \\ 1 & if \bowtie = >, \\ 0 & if \bowtie = \simeq. \end{array} \right.$

Obviously, the above definition is related to the grading interpretation, except that when $p \gg q$, we define diff(p,q) = 2 instead of diff $(p,q) \ge 2$ for simplicity. Here we do not directly define diff(p,q) = f(p) - f(q) because Q is not necessarily consistent and thus the grading interpretation f does not necessarily exist.

We begin with the study of inconsistency between propositions like $p \bowtie_1 q$ and $p \bowtie_2 q$. We call this kind of inconsistency as **redundancy of** p **w.r.t** q, and denote it as $\mathsf{Rdu}(p:q)$ (Hence $\mathsf{Rdu}(p:q)$ is not equal to $\mathsf{Rdu}(q:p)$).

Recall that we use $E(n) = (Q_0, \dots, Q_n)$ to denote a prioritized QCB $(\cup E)$ constructed from E up to n times application of transition rules. From Definition 8, we can

get at most the following three propositions that relate symbol p with symbol q with different priority levels, $(p \gg q, a_{\gg}), (p > q, a_{>}), (p \simeq q, a_{\simeq}) \in E(n)$ where a_{\bowtie} is the priority level of the corresponding proposition, for $\bowtie = \gg, >, \simeq$.

Based on Definition 10, we define $\mathsf{Rdu}(p:q)$ as follows:

$$\mathsf{Rdu}(p:q) = \max_{i \in I_{\bowtie}}(\mathsf{diff}_{i}(p,q)) - \min_{j \in I_{\bowtie}}(\mathsf{diff}_{j}(p,q)) \tag{1}$$

Here I is the set of the propositions with minimal priority level, e.g. if $a_{\gg} = a_{\simeq} < a_{>}$, then $I = \{p \gg q, p \simeq q\}$, and I_{\bowtie} is the set of comparison connectives appeared in I. It should be noted that if $a_{\gg} = a_{\simeq} = a_{>} = +\infty$, then $I = \emptyset$ and $\mathsf{Rdu}(p:q)$ is undefined.

The semantics of Equation (1) is clear. If there are different observations with different strengthes of preference on p to q, then the inconsistency of between p and q comes out as the largest difference between preference strengthes.

Example 6 If $(p \gg q, 3), (p > q, 3), (p \simeq q, +\infty) \in E(n)$, then since $3 = 3 < +\infty$, the index set I is $I = \{p \gg q, p > q\}$ (and $I_{\bowtie} = \{\gg, >\}$). Therefore, we have

$$\mathsf{Rdu}(p:q) = \mathsf{diff}_{\gg}(p,q) - \mathsf{diff}_{>}(p,q) = 2 - 1 = 1.$$

Notice that if |I| = 1, then we have $\mathsf{Rdu}(p:q) = 0$ which implies there is no redundancy of p w.r.t. q. This result is intuitive, since if the priority level of one proposition is smaller than that of the other two, then this proposition is more reliable than them, e.g. $a_{\gg} < \min(a_{>}, a_{\simeq})$, then we should conclude that $p \gg q$ is more reliable than p > q and $p \simeq q$, and it gives no redundancy or inconsistency.

Next, let us consider the inconsistency between propositions like $p \bowtie_1 q$ and $q \bowtie_2 p$. We call this kind of inconsistency as **conflict between** p **and** q and denote it as lnc(p:q).

Based on Definition 8, there will be at most three priority levels, $a_{\gg}, a_{>}, a_{\sim}$, for three propositions $(p \gg q)$, (p > q), and $(p \simeq q)$, respectively. Similarly, there will be at most three values $b_{\gg}, b_{>}, b_{\simeq}$ for $(q \gg p)$, (q > p), and $(q \simeq p)$, respectively. The priority levels will also play an important role here. In fact, only when $min(a_{\gg}, a_{>}, a_{\simeq}) = min(b_{\gg}, b_{>}, b_{\simeq}) < +\infty$, it is meaningful to measure the inconsistency (conflict) between p and q. If $min(a_{\gg}, a_{>}, a_{\simeq}) = min(b_{\gg}, b_{>}, b_{\simeq}) = +\infty$, the inconsistency analysis is meaningless. If $min(a_{\gg}, a_{>}, a_{\simeq}) \neq min(b_{\gg}, b_{>}, b_{\simeq})$, then the conflict between p and q is defined as 0 since the priority levels of these propositions already exclude the inconsistencies between propositions.

Now, we concentrate on the situation where $min(a_{\gg}, a_{>}, a_{\simeq}) = min(b_{\gg}, b_{>}, b_{\simeq}) < +\infty$, where a_{\bowtie} is for $p \bowtie q$ and b_{\bowtie} is for $q \bowtie p$. We define the conflict between p and q as follows.

$$\operatorname{Inc}(p:q) = \min_{i \in I^1_{\bowtie}}(\operatorname{diff}_i(p,q)) + \min_{j \in I^2_{\bowtie}}(\operatorname{diff}_j(q,p))$$
(2)

Here I_{\bowtie}^1 (resp. I_{\bowtie}^2) is the set of comparison connectives for propositions with the minimal priority level among $(p \gg q)$, (p > q), and $(p \simeq q)$ (resp. $(q \gg p)$, (q > p), and $(q \simeq p)$).

The semantics of Equation (2) is rather intuitive. If one observation gives that p is preferred to q while another observation provides q is preferred to p, then obviously it

leads to inconsistency. The degree of inconsistency (conflict) can be measured as a *distance* between the two observations. Equation (2) captures this intuition and provides a *minimal* distance between the two observations, in terms of two propositions.

Example 7 If $(p \gg q, 3), (p > q, 3), (p \simeq q, +\infty), (q \gg p, 3), (q > p, 5), (q \simeq p, +\infty) \in E(n)$, then $I^1_{\bowtie} = \{\gg, >\}$ and $I^2_{\bowtie} = \{\gg\}$. Therefore, we get

$$\mathrm{Inc}(p:q) = \mathrm{diff}_{>}(p,q) + \mathrm{diff}_{\gg}(q,p) = 1+2 = 3.$$

From Proposition 3, we know that $a_{\simeq} = b_{\simeq}$, thus if they are the minimal ones among a_{\bowtie} and b_{\bowtie} where $\bowtie \in \{\gg, >, \simeq\}$, then from Equation (2), we have

$$lnc(p:q) = diff_{\simeq}(p,q) + diff_{\simeq}(p,q) = 0 + 0 = 0$$

which implies there is no inconsistency (conflict) between p and q. This result is intuitive and consistent with common sense reasoning, since if $a_{\simeq} = b_{\simeq}$ have the minimal priority level, then it implies that $p \simeq q$ and $q \simeq p$ are more reliable than any other propositions relating to p and q. Furthermore, $p \simeq q$ and $q \simeq p$ are obviously consistent.

Proposition 4 Given a QCB Q, let p and q be two symbols and lnc(p : q) is defined, then we have lnc(p : q) = 0 or $lnc(p : q) \ge 2$.

5 Merging Multiple QC Bases

In this section, we discuss the merging of multiple QCBs.

We consider three kinds of merging operators. The first two kinds are syntax-based, of which the first kind of operators intends to obtain a consistent result and the second kind only pays attention to pair-wise comparison relations no matter whether the overall merged result is consistent. This might give the impression that pair-wise merging seems rather counterintuitive at the first glance, but as the QC relations are not precise relations themselves and in many settings, only the pair-wise comparison results are considered, e.g., when querying the comparison relation between two drugs. There-fore, pair-wise based merging is useful. In addition, human's beliefs are themselves not always consistent. Thus, pair-wise based merging deserves an investigation. The third kind of merging operators follows a similar way of model based merging used for merging knowledge bases [17]. It makes use of grading interpretations and takes the grading interpretation of the largest consistent subset of QCBs as the grading interpre-tation of the merged result.

5.1 Consistency-based merging

In this subsection, we investigate merging operators that produce a consistent result.

Definition 11 A sub-consistent QCP is a QCP E such that for each $Q \in E$, Q is consistent based on Definition 4.

Definition 12 A consistent QC-merging operator Δ is a function from the set of all QCPs to the set of all sub-consistent QCPs.

Recall that from a QCP E, a list of prioritized QCBs $E(n) = (Q_0, \ldots, Q_n)$ can be constructed. Let Cons(E) denote the set of consistent subsets of E(n) such that $\forall S = (S_0, \ldots, S_n) \in Cons(E), \bigcup S = \bigcup_{i=0}^n S_i$ is consistent.

In [9], some merging operators are mentioned for merging prioritized knowledge (observation) bases. Here we propose some similar operators for merging prioritized QCBs, i.e., E(n)s.

If \succ is a strict order (i.e. a transitive and asymmetric binary relation) on a set X, then for any $Y \subseteq X$ we denote by $Max(\succ, Y)$ the set of undominated elements of Y with respect to \succ [9], i.e.,

$$Max(\succ, Y) = \{ y \in Y | \not\exists z \in Y, \ s.t., \ z \succ y \}.$$

Definition 13 (discrimin, [4], [23], [3]) For $S, S' \in Cons(E)$, define $S' \succ_{discrimin} S$ iff $\exists k$ such that

1. $\bigcup_{j=0}^{k} Q_j \bigcap S' \supset \bigcup_{j=0}^{k} Q_j \bigcap S$, and

2. for all
$$i < k$$
, $\bigcup_{j=0}^{i} Q_j \cap S' = \bigcup_{j=0}^{i} Q_j \cap S$.

Then $\Delta_{discrimin}(E) =$

$$\left\{ \left(\ \right) S, S \in Max(\succ_{discrimin}, Cons(E)) \right\}$$

Definition 14 (leximin, [3], [15]) For $S, S' \in Cons(E)$, define $S' \succ_{leximin} S$ iff $\exists k$ such that

1. $|\bigcup_{j=0}^{k} Q_j \cap S'| > |\bigcup_{j=0}^{k} Q_j \cap S|$, and

2. for all
$$i < k$$
, $|\bigcup_{j=0}^{i} Q_j \cap S'| = |\bigcup_{j=0}^{i} Q_j \cap S|$

Then $\Delta_{leximin}(E) =$

$$\left\{ \left[\ \right] S, S \in Max(\succ_{leximin}, Cons(E)) \right\}$$

The results of $\Delta_{discrimin}(E)$ and $\Delta_{leximin}(E)$ are in fact QCPs. In addition, for simplicity and convenience, we hereafter always remove the propositions like $p \simeq p$, etc, from the result QCPs.

Example 8 Let $E = \{\{p > q\}, \{r > p\}, \{q \simeq r, r \simeq q\}, \{p \simeq p, q \simeq q, r \simeq r\}\},\$ then $E(n) = (Q_0, \ldots, Q_n)$ where $Q_0 = \{p > q, r > p, q \simeq r, r \simeq q, p \simeq p, q \simeq q, r \simeq r\},\ Q_1 = \{p > r, r \gg q, q > p\},\ Q_2 = \{p \gg p, r \gg r, q \gg q, q \gg r, r \gg p, p \gg q\},\$ and $Q_3 = \{p \gg r, q \gg p\}$ and for any $i > 3, Q_i = \emptyset$.

Using the discrimin merging operator, we get: $Max(\succ_{discrimin}, Cons(E)) = \{S_1, S_2, S_3\}$ where $S_1 = (\{p > q, q \simeq r, r \simeq q\}, \{p > r\}), S_2 = (\{r > p, q \simeq r, r \simeq q\}, \{q > p\}), and S_3 = (\{p > q, r > p\}, \{r \gg q\}).$ Thus, $\Delta_{discrimin}(E) = \{\{p > q, q \simeq r, r \simeq q, q > r\}, \{r > q, r > p, r \gg q\}\}$.

Using the leximin merging operator, similarly we get: $\Delta_{leximin}(E) = \{\{p > q, q \simeq r, r \simeq q, p > r\}, \{r > p, q \simeq r, r \simeq q, q > p\}\}.$

A merged result using either $\Delta_{discrimin}(E)$ or $\Delta_{leximin}(E)$ may contain more than one consistent subset. That is, the consistent-based merging in fact extracts the largest consistent subsets of QCPs. This approach hence lists all the possible alternatives of the QC relations between drugs. Therefore, it provides an all-sided view on those drugs and is particularly suitable for proving information to medical scientists for further investigation.

It should be pointed out that since all Q_i s are simeq closed, i.e. the two propositions $p \simeq q$ and $q \simeq p$ either both are appeared in Q_i or none of them are, and a grading interpretation will always satisfy/dissatisfy $p \simeq q$ and $q \simeq p$ simultaneously. So when we use the leximin operator, it would be more meaningful that propositions $p \simeq q$ and $q \simeq p$ together should only be counted once (not twice). Doing so, the leximin operator will obtain the same result as the discrimin operator in the above example (this is not true in general).

The following simple example show why $p \simeq q$ and $q \simeq p$ together should only be counted once in the leximin operator.

Example 9 Let one agent believe that $p \simeq q$ and another agent believe that p > q, then it is reasonable to consider that these two propositions compete and each one do not prevail another. But when we use the leximin operator on the simeq closed version of $E = \{\{p \simeq q, q \simeq p\}, \{p > q\}, \{p \simeq p, q \simeq q\}\}$, we will get $\Delta_{leximin}(E) = \{p \simeq q, q \simeq p, p \simeq p, q \simeq q\}$ which shows p > q is neglected. Obviously it is not intuitive, and it is because the cardinality of $\{p \simeq q, q \simeq p\}$ is 2 while the cardinality of $\{p > q\}$ is 1. So if we want to solve this problem, $p \simeq q$ and $q \simeq p$ should only be counted once.

5.2 Pair-wise based merging

Definition 15 A pair-wise QC-merging operator Δ is a function from the set of all QCPs to the set of QCBs.

For a given QCP E and the induced prioritized E(n), we consider all the QC relations between each pair of symbols p and q appearing in E. Of course, p and q should be absolutely accessible based on Definition 9 or relatively accessible if the threshold is given.

Suppose a pair of symbols p and q are accessible, if the inconsistency between p and q (see Equation (2)) is large, i.e., different observations differ strongly on the preferences of p and q, then cautiously, we will not produce any QC relation between p and q.

Formally, suppose p and q are accessible, with the help of Proposition 4, we define the pair-wise based merging operator as the output of the following algorithm (Recall a_{\bowtie} is the priority level of $p \bowtie q$ and b_{\bowtie} is that for $q \bowtie p$).

Algorithm Pair-wise based Merging Begin Input: a simeq closed QCP EOutput: a merged result denoted as $\Delta_{pw}(E)$ $\Delta_{pw}(E) = \emptyset$; for each symbol pair $p \neq q$ if $(\ln c(p:q))$ is not defined

if $min(a_{\gg}, a_{>}, a_{\simeq}) < min(b_{\gg}, b_{>}, b_{\simeq})$ if $a_{\gg} > a_{>}$ then $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{p \gg q\}$ else $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{p > q\}$ else if $b_{\gg} > b_>$ then $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{q \gg p\}$ else $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{q > p\}$ else if lnc(p:q) = 0if $a_{\simeq} = min(a_{\gg}, a_{>})$ and $b_{\simeq} = min(b_{\gg}, b_{>})$ then $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{p \simeq q, q \simeq p\}$ else if $a_{\simeq} < min(a_{\gg}, a_{>})$ and $b_{\simeq} < min(b_{\gg}, b_{>})$ then $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{p \simeq q, q \simeq p\}$ else if $a_{\simeq} < min(a_{\gg}, a_{>})$ and $b_{\simeq} = b_{\gg}$ then $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{q > p\}$ else if $a_{\simeq} = a_{\gg}$ and $b_{\simeq} < min(b_{\gg}, b_{>})$ then $\Delta_{pw}(E) = \Delta_{pw}(E) \cup \{p > q\}$

End

We denote the above merging operation as Δ_{pw} . To demonstrate the above algorithm, we discuss the following cases.

- lnc(p:q) is defined and lnc(p:q) ≥ 2 This shows that the preference of p and q differs very strongly. Thus we cautiously do not give the merged QC relation between p and q.
- lnc(p:q) is defined and lnc(p:q) = 0 Here we have a₂ ≤ min(a_≫, a_>) and b₂ ≤ min(b_≫, b_>). This situation has two subcases.
 - (a) If a₂ = min(a_≫, a_>) and b₂ = min(b_≫, b_>), then some observations prefer p, some prefer q, and some take p and q equivalently, thus naturally we provide the merged relation as p ≥ q.
 - (b) Either a₂ < min(a_≫, a_>) or b₂ < min(b_≫, b_>) holds. If they both hold, then obviously we should let p 2 q as the merged result as p 2 q is the dominated point of view. If only one of them holds, e.g., b₂ < min(b_≫, b_>) holds while a₂ = min(a_≫, a_>), then if we have a_≫ = min(a_≫, a_>), it implies that some observations significantly prefer p to q(p ≫ q) while some others treat them indifferently (p 2 q). Hence we may averagely take the QC relation between p and q as p > q, else if we have a₂ = a_> < a_≫, it implies that some observations prefer p to q while some treat them indifferently, a cautious view will not give any merged QC relation between p and q.
- 3. lnc(p:q) is undefined.

Without loss of generality, suppose $min(a_{\gg}, a_{>}, a_{\simeq}) < min(b_{\gg}, b_{>}, b_{\simeq})$ and $a_{\simeq} > min(a_{\gg}, a_{>})$. If $a_{\gg} < a_{>}$ or $a_{<} > a_{\gg}$, obviously we can conclude p > q or $p \gg q$, respectively. If $a_{\gg} = a_{>}$, we will cautiously give the merged relation as p > q.

Example 10 Let $E = \{\{p > q\}, \{r > q\}, \{q \simeq s, s \simeq q\}, \{q > p\}, \{p > r\}, \{r \gg s\}, \{p \simeq p, q \simeq q, s \simeq s, r \simeq r\}\}$, then $E(n) = (Q_0, \dots, Q_n)$ where $Q_0 = \{p > q, r > q, q \simeq s, s \simeq q, q > p, p > r, r \gg s, p \simeq p, q \simeq q, s \simeq s, r \simeq r\}$, $Q_1 = \{p > s, p \gg p, r > s, r \gg p, q \gg q, q \gg r, p \gg s, r \gg q, s > p\}$, $Q_2 = \{p \gg r, r \gg r, q \gg p, q \gg s, s \gg q, s \gg p, s \gg r\}$ and for $i \ge 3$, $Q_i = \emptyset$. Using the pair-wise merging, we get: $\Delta_{pw}(E) = \{p > r, r > q, q \simeq s, r \gg s\}$.

Pair-wise based merging is particularly suitable for the query situation. In this situation, scientists are solely interested in the comparison relation between two drugs which can be nicely answered by pair-wise merging when these two drugs are considered as a pair.

5.3 Grading interpretation based merging

There are two main categories of approaches to merging knowledge bases, syntaxbased and model-based. The consistency-based merging and pair-wise based merging defined above are to some extent syntax-based merging methods. In this section, we propose a grading interpretation based merging method which can be understood as model-based merging.

Let $E = \{Q_1, \ldots, Q_n\}$ be a QCP, and let $E' = \{S_1, \ldots, S_n\}$ be any consistent QCP such that $S_i \subseteq Q_i$. Let T_E be the set of all such E's. We define a total pre-order relation \geq_{T_E} on T_E as $E^1 \geq_{T_E} E^2$ iff $\sum_{i=1}^n |S_i^i| \geq \sum_{i=1}^n |S_i^2|$, where $E^1, E^2 \in T_E$. The grading interpretation based merging method will choose $E^* \in T_E$ such that

The grading interpretation based merging method will choose $E^* \in T_E$ such that $E^* \ge_{T_E} E'$ for $\forall E' \in T_E$. Since E^* is consistent, from Theorem 2, it has a unique minimal grading interpretation f^* .

With this grading interpretation f^* , we are able to get the QC relations between each pair of symbols with respect to Definition 3. Namely, we can define a mapping gfrom the set of all f^* to the set of QCBs. Thus we get a consistent merging result.

Formally, we define the operator as follows.

$$\Delta_{gi}(E) = \{g(f^*) : \exists E^* \in max(T_E, \leq_{T_E})\},\$$

where f^* is the unique minimal grading interpretation of E^* .

Note that this merging operator does not need a QCP E to be simeq closed because it does not use the induction rules.

Example 11 Let $E = \{\{p > q\}, \{r > q\}, \{q \simeq s, s \simeq q, p > q\}, \{q > p\}, \{p > r, r > q\}, \{r \gg s\}\}$, then we get $E^* = \{\{p > q\}, \{r > q\}, \{q \simeq s, s \simeq q, p > q\}, \emptyset, \{r > q\}, \emptyset\}$. f* is such that:

$$f^*(s) = f^*(q) = 0, f^*(p) = f^*(r) = 1.$$

Thus we have the merged QCB $Q = \{p > q, r > q, p > s, r > s, p \simeq r, r \simeq p, q \simeq s, s \simeq q\}.$

Grading interpretation based merging gives a unique and consistent result. So it is very useful to help clinicians to make decisions over various drugs.

In summary, consistent-based merging is a nice choice for medical scientists who want to get a comprehensive opinion, pair-wise based merging is especially applicable for query situations, and grading interpretation based merging is most suitable for the clinicians making decision.

6 Case Study

There are many clinical trials on evaluating and comparing the effects of different drugs on lowering intraocular pressure (IOP for short). Here we consider some of the research papers reporting such trials and use the modeling and merging methods introduced in the paper to analyze qualitative comparison knowledge appeared in these papers.

Let tr denote the effect of travoprost for IOP reduction, la for latanoprost, ti for timolol, and bi for bimatoprost.

In [1], it states *IOP reduction obtained with travoprost was significantly higher than that obtained with latanoprost.* Thus we have the first QC base as $Q_1 = \{tr \gg la\}$.

In [14], it concludes According to data available, travoprost is more effective than timolol in lowering IOP in patients with open-angle glaucoma or ocular hyperextension. Compared with other prostaglandin analogues, travoprost appears to be equivalently to bimatoprost and latanoprost. So, we have the 2nd QC base as $Q_2 = \{tr > ti, tr \simeq bi, tr \simeq la\}$.

In [12], it summarizes Patients on travoprost therapy showed lower mean IOP levels than those on latanoprost. Similarly, we get the 3rd QC base $Q_3 = \{tr > la\}$.

In [7], it draws a conclusion as Patients treated with travoprost and bimatoprost had lower IOP levels at the end of follow-up than those treated with latanoprost. Thus we have the 4th QC base $Q_4 = \{tr > la, bi > la\}$.

In [6], it states *travoprost was superior to latanoprost*, and we get the 5th QC base $Q_5 = \{tr > la\}.$

Based on these five QC bases, we get a QCP $E = \{Q_1, ..., Q_5\}$. The simeq closed version of this QCP is $E = \{\{tr \gg la\}, \{tr > ti, tr \simeq bi, tr \simeq la, bi \simeq tr, la \simeq tr\}, \{tr > la\}, \{tr > la, bi > la\}, \{tr > la\}, \{tr \simeq tr, la \simeq la, ti \simeq ti, bi \simeq bi\}\}$ and E(n) = (Q(0), Q(1), Q(2), ...) where $Q(0) = \{tr \gg la, tr > ti, tr \simeq bi, tr \simeq la, bi \simeq tr, la \simeq tr, tr > la, tr > la, bi > la, tr > la, tr > tr, la \simeq la, ti \simeq ti, bi \simeq bi\}, Q(1) = \{tr \gg tr, bi \gg la, bi > ti, la \gg la, la > ti, la > la, tr > tr, bi \gg la, la > ti, la > bi, bi \gg bi, bi \gg tr, bi \gg tr, la \gg tr, la \gg tr, la \gg ti, la > bi\}$ and for i > 2, we have $Q(i) = \emptyset$.

Using the merging operators introduced in the previous section, we have

 $\Delta_{discrimin} = \{\{tr > ti, tr \gg la, tr \simeq bi, bi \simeq tr, bi \gg la, bi > ti\}, \{tr > ti, tr > la, tr \simeq bi, bi \simeq tr, bi > la, bi > ti\}, \{tr > ti, tr \simeq la, la \simeq tr, tr \simeq bi, bi \simeq tr, bi > ti, la > ti\}, \{tr > ti, tr \simeq la, la \simeq tr, tr \simeq bi, bi \simeq tr, bi \simeq tr, bi > ti, la > ti\}, \{tr > ti, tr \simeq la, la \simeq tr, tr \simeq bi, bi \simeq tr, bi > ti, la > ti\}, \{tr > ti, tr > ti,$

 $\Delta_{leximin} = \{\{tr > ti, tr > la, tr \simeq bi, bi \simeq tr, bi > la, bi > ti\}\},\$

 $\Delta_{pw} = \{ tr > ti, tr > la, tr \simeq bi, bi \simeq tr, bi > la, bi > ti, la > ti \},\$

 $\Delta_{qi} = \{ tr > ti, tr > la, tr \simeq bi, bi \simeq tr, bi > la, bi > ti, la \simeq ti, ti \simeq la \}.$

By examining these different merged results, we can see that the $\Delta_{discrimin}$ produces the more sophisticated result than all the other three methods. The $\Delta_{discrimin}$

points out that there may be three kinds of possible merging results, but it cannot tell which result is more plausible. Nevertheless, $\Delta_{leximin}$, Δ_{pw} and Δ_{gi} operators give us deterministic results, moreover, these results are similar (but not entirely the same). Intuitively, it seems that the results of $\Delta_{leximin}$ and Δ_{pw} are more reasonable and applicable as the gi merging operator gives $la \simeq ti$, $ti \simeq la$ which has no support from original QC bases and the result of $\Delta_{discrimin}$ is too complicated.

Let us take for example the comparison between tr and la. The five original QC bases together contain three possible propositions for these two symbols. In the three different results from $\Delta_{discrimin}$, each of these propositions is included in one of its resulting subsets. However, proposition tr > la appears in all the results from $\Delta_{leximin}$, Δ_{pw} and Δ_{gi} which once again shows the consistency of these operators and also that propositions for symbols tr and la.

7 Conclusion

In this paper, we systematically studied how to model and reason with qualitative comparison knowledge and proposed three kinds of propositions (qualitative comparison relations) to model such knowledge i.e. \gg , > and \simeq . We formalized some basic definitions as QC language, QCB, and QCP, etc. In addition, we proposed some induction rules for the QC relations. We also gave grading interpretations to the QC relations and their induction rules. With the help of grading interpretation, we can define whether a QCB or a QCP is consistent. Moreover, for inconsistent QCPs, we provided measures of inconsistency and proposed three types of merging operators. Some merging operators are similar to the ones in prioritized knowledge base merging while others use either a pair-wise based method or are based on the grading interpretations to perform merging.

A report on a clinical trial usually presents quantitative knowledge as well as qualitative knowledge. Quantitative knowledge is typically studied by systematic reviews or meta-analysis [19, 20, 21], etc. The main problem of analyzing quantitative knowledge is that it is time-consuming and often it should be done by an expert and with a specialized software. However, even experts cannot scale well with the increasing rate of publication of new results, as it requires a substantial amount of painstaking work. So frequently qualitative knowledge about detailed (quantitative) information is used to get a summaritive conclusion of the trial study. In addition, some clinical trials reports only provide qualitative knowledge without giving quantitative information whilst there is no clinical trial report that only reports quantitative knowledge without any qualitative summary throughout our study. Therefore, an efficient and easy-to-use framework on analyzing qualitative knowledge is appealing.

Although we have tried to give a thorough study on this topic, there are still many issues to be investigated in the future. One problem is that when we consider the cautious use of transition rules, we in fact treat the transition rules equally. But in fact some of the induction rules can be taken for granted and thus be used without caution. For example, if $p \gg q$ and $q \gg r$, then it can induce $p \gg r$ without controversy. Therefore the transition rules might be partitioned based on their *rightness*. Further-

more, a detailed comparison with our framework with order of magnitude reasoning is also attractive .

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Appendix

Proof of Proposition 1: If $q \simeq p$ or q > p or $q \gg p$, then from the transition rule and $p \gg r$, we immediately get $q \gg r$ which contradicts to q > r. Therefore, according to the *totality* rule, we have p > q or $p \gg q$.

Proof of Theorem 1:

Reflection $p \simeq p$. Since f(p) - f(p) = 0, we have $p \simeq p$.

- **Symmetry** $p \simeq q$ implies $q \simeq p$. If $p \simeq q$, then f(p) - f(q) = 0, thus f(q) - f(p) = 0, which implies $q \simeq p$.
- **Totality** $p \simeq q$ or q > p or p > q or $p \gg q$ or $q \gg p$. If f(p) - f(q) = 0, then $p \simeq q$, if |f(p) - f(q)| = 1, then p > q or q > p, if $|f(p) - f(q)| \ge 2$, then $p \gg q$ or $q \gg p$.
- **Transition** 1. $p \bowtie_1 q$ and $q \bowtie_2 r$ implies $p \gg r$ if at least one of \bowtie_1 and \bowtie_2 is \gg . If $\bowtie_1 = \gg$, then we have $f(p) - f(q) \ge 2$ and $f(q) - f(r) \ge 0$, thus $f(p) - f(r) \ge 2$ which implies $p \gg r$. If $\bowtie_2 = \gg$, the proof is similar.
 - 2. $p \bowtie q$ and $q \simeq r$ implies $p \bowtie r$. From $q \simeq r$, we have f(q) - f(r) = 0, thus f(p) - f(r) = f(p) - f(q) + f(q) - f(r) = f(p) - f(q). Therefore $p \bowtie q$ implies $p \bowtie r$.
 - 3. $p \simeq q$ and $q \bowtie r$ implies $p \bowtie r$. Similar to the above.
 - 4. p > q and q > r implies $p \gg r$. From p > q and q > r, we have f(p) - f(q) = 1 and f(q) - f(r) = 1, then we have f(p) - f(r) = 2 which implies $p \gg r$.

Proof of Theorem 2: We will construct a minimal grading interpretation and show it is the unique one. Let f^* be a mapping from \mathcal{P} to the set of ordinal numbers and it satisfies the following

$$f^*(p) = \min_{f \in \mathcal{F}} f(p).$$

Obviously, as $\forall p \in \mathcal{P}$, $f(p) \ge 0$, we have $f^*(p) \ge 0$. First we show that $f^* \in \mathcal{F}$. It suffices to show the following:

1. If $p \simeq q$, then $f^*(p) - f^*(q) = 0$. As $p \simeq q$, then $\forall f \in \mathcal{F}$, we have f(p) - f(q) = 0, or f(p) = f(q), thus we have

$$f^*(p) = \min_{f \in \mathcal{F}} f(p) = \min_{f \in \mathcal{F}} f(q) = f^*(q).$$

2. If p > q, then $f^*(p) - f^*(q) = 1$. As p > q, then $\forall f \in \mathcal{F}$, we have f(p) = f(q) + 1, thus we have

$$f^*(p) = \min_{f \in \mathcal{F}} f(p) = \min_{f \in \mathcal{F}} f(q) + 1$$
$$= \min_{f \in \mathcal{F}} f(q) + 1 = f^*(q) + 1.$$

3. If $p \gg q$, then $f^*(p) - f^*(q) \ge 2$. As $p \gg q$, then $\forall f \in \mathcal{F}$, we have $f(p) \ge f(q) + 2$, thus we have

$$f^*(p) = \min_{f \in \mathcal{F}} f(p) \ge \min_{f \in \mathcal{F}} f(q) + 2$$
$$= \min_{f \in \mathcal{F}} f(q) + 2 = f^*(q) + 2.$$

So f^* is a grading interpretation, as $\forall f \in \mathcal{F}$ and $\forall p \in \mathcal{P}$, we have $f^*(p) \leq f(p)$, hence $\sum_{p \in \mathcal{P}} f^*(p) \leq \sum_{p \in \mathcal{P}} f(p)$, therefore f^* is a minimal grading interpretation.

Let g be any minimal grading interpretation, as $f^*(p) \leq g(p)$ for $\forall p \in \mathcal{P}$ and $\sum_{p \in \mathcal{P}} g(p) = \sum_{p \in \mathcal{P}} f^*(p)$ (the definition of minimal grading interpretation), it should be $f^*(p) = g(p)$ for $\forall p \in \mathcal{P}$. It shows that g is f^* , so f^* is the unique minimal grading interpretation.

Proof of Proposition 2: Suppose a proposition $p \simeq q \in Q$, we need to show that $q \simeq p$ is also in Q. From the construction process of Q, the proposition $p \simeq q$ should be induced by two propositions $p \simeq r$ and $r \simeq q$ using the transition induction rules. Since Q_0 is simeq closed, Q_0 contains $p \simeq r$ and $r \simeq q$ implies Q_0 also contains $r \simeq p$ and $q \simeq r$, which can induce $q \simeq p$ by the transition induction rules. This finishes the proof.

Proof of Proposition 3: If $p \simeq q$ has priority level $+\infty$, i.e. $p \simeq q \notin Q_i$, $\forall 0 \le i \le n$, then from Proposition 2, as Q_i is simeq closed, it should be $q \simeq p \notin Q_i$, $\forall 0 \le i \le n$. Hence $q \simeq p$ also has priority level $+\infty$. If $p \simeq q$ has priority level k, i.e. $p \simeq q \notin Q_i$, $\forall 0 \le i < k$ and $p \simeq q \in Q_k$, then still from Proposition 2, we know $q \simeq p \notin Q_i$, $\forall 0 \le i < k$ and $q \simeq p \in Q_k$ which implies that $q \simeq p$ has priority level k.

Proof of Proposition 4: If lnc(p:q) is defined, then we have $min(a_{\gg}, a_{>}, a_{\simeq}) = min(b_{\gg}, b_{>}, b_{\simeq}) < +\infty$. If $a_{\simeq} = min(a_{\gg}, a_{>}, a_{\simeq})$, then from Proposition 3, it should be

$$b_{\simeq} = a_{\simeq} = \min(a_{\gg}, a_{>}, a_{\simeq}) = \min(b_{\gg}, b_{>}, b_{\simeq}),$$

therefore $\operatorname{Inc}(p:q) = 0 + 0 = 0$. If $b_{\simeq} = \min(b_{\gg}, b_{>}, b_{\simeq})$, similarly $\operatorname{Inc}(p:q) = 0 + 0 = 0$. If neither $a_{\simeq} = \min(a_{\gg}, a_{>}, a_{\simeq})$ nor $b_{\simeq} = \min(b_{\gg}, b_{>}, b_{\simeq})$, then we have $\operatorname{Inc}(p:q) \ge 1 + 1 = 2$.

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