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Adaptive Merging of Prioritized Knowledge Bases

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Abstract. In this paper, we propose an adaptive algorithm for merging *n* ($n \geq 2$) prioritized knowledge bases which takes into account the degrees of conflict and agreement among these knowledge bases. The algorithm first selects *largely partially maximal consistent subsets (LPMCS)* of sources by assessing how (partially) consistent the information in the subset is. Then within each of these created subsets, a *maximal consistent subset* is further selected and knowledge bases in it are merged with a suitable conjunctive operator based on the degree of agreement among them. This result is then merged with the remaining knowledge bases in the corresponding LPMCS in the second step through the relaxation of the minimum operator. Finally, the knowledge bases obtained from the second step are merged by a maximum operator. In comparison with other merging methods, our approach is more context dependent and is especially useful when most sources of information are in conflict.

Keywords: Possibilistic logic, prioritized knowledge bases, belief merging, context dependent merging

1. Introduction

Merging multiple sources of information is important in many areas, such as sensor data fusion (e.g., [AG92]) and database integration (e.g., [LS98, Rev97]). This process becomes more complex when uncertainty exists. Possibilistic logic provides a good logical framework for modelling and reasoning with uncertainty. In this framework, uncertain information from a sources is represented as a *possibilistic knowledge base*, which is a set of weighted formulas. A possibilistic knowledge base has a unique

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possibility distribution associated with it. In [BDP97, BDKP02], some semantic merging operators were proposed to aggregate *possibility distributions* of original possibilistic knowledge bases, the result is a new possibility distribution. Then the syntactical counterpart of a semantic merging operator is applied to the possibilistic bases, and the result of merging is a possibilistic knowledge base whose possibility distribution is the one obtained by the semantic merging operator. The merging operators proposed so far (e.g., [BDP97, BDKP02]) can be divided into two classes, one class contains conjunctive operators (for example, the minimum operator) and the other consists of disjunctive operators (for example, the maximum operator). The advantage of the conjunctive operators is that when original knowledge bases are consistent, they exploit their complementarities by recovering all the symbolic information. However, the conjunctive operator is not advisable to merge knowledge bases which are highly conflicting because the *inconsistency degree* of the resulting knowledge base can be very high. In contrast, the disjunctive operators are appropriate to merge inconsistent knowledge bases because the resulting knowledge base is consistent as long as one of the original knowledge bases is consistent. The disadvantage of disjunctive operators is that too much information is lost after merging. Because of the pros and cons of the conjunctive and disjunctive operators, it is not advisable to use only one of them when information from multiple sources partially agrees with each other and only some of these sources are reliable. Several adaptive merging rules have been proposed to integrate both the conjunctive and disjunctive operators (e.g., [DPT88, DP92, DP94, DFP00, DP01]).

In [DPT88], an adaptive operator was proposed which considered j sources out of all the sources, where it was assumed that there were j sources reliable. Since it was not known which j sources were reliable, all the subsets with cardinality j were considered and sources in each of these subsets were merged conjunctively. The major problem with this method is that some of the subsets may contain sources which are in conflict, so it is not appropriate to merge them conjunctively. Another drawback is that it only utilizes two operators, one is a conjunctive operator (e.g., the minimum operator) and the other is a disjunctive operator (e.g., the maximum operator). There are many other conjunctive and disjunctive operators available that may be more appropriate to different subsets of sources, according to the degrees of conflict and agreement among sources within a subset [QLB05].

Another adaptive rule was proposed in [DP92] which also utilizes the maximum and the minimum operators. The rule was initially proposed to merge two sources only, and was later extended to merge more than two sources in [DP94]. However, it is computationally very expensive because it needs to compute all the maximal consistent subsets, which is known to be a very difficult task. Moreover, when most possibilistic knowledge bases are in conflict, merging them using this rule may either delete too much information or the rule is reduced to the disjunctive merge mode (e.g., using the maximum operator only).

In summary, none of the adaptive rules available so far is satisfactory for merging multiple sources of information that is partially consistent and many sources are involved in conflict. In this paper, we attempt to investigate how merging such information can be more context-dependent and how various operators (not just the maximum and minimum) can be integrated into the fusion process. We especially consider the following issues in this paper to utilize a context-dependent adaptive merging.

- the selection of either a conjunctive operator or a disjunctive operator should include the measures of the quality of merged information in both modes and the level of information loss in a disjunctive mode;

- given a knowledge base that is believed to be of high quality and can be taken as a reference, other knowledge bases should be ordered based on their degrees of consistency with this reference knowledge base. In this way, a subset of sources can be formed around this reference which should guarantee that the sources in the subset are consistent to at least a certain degree;

- to reinforce the beliefs from consistent sources, suitable reinforcement operators may be used instead of the minimum operator. The selection criteria of a reinforcement operator should address the degree of strong agreement among multiple knowledge bases;
- multiple operators, including maximum, minimum, and reinforcement, should be integrated into merging at different stages to deal with different subsets of sources.

The end result of the investigation of above issues has led to the design of a context-dependent adaptive algorithm for merging *n* ($n \geq 2$) prioritized knowledge bases. The algorithm first selects *largely partially maximal consistent subsets* (LPMCSs) of sources and then further selects a *maximal consist subset* within each of the LPMCSs. Different merging operators are applied to these different subsets based on the nature of the subset (e.g., maximal consistent or largely partially maximal consistent). We believe that this algorithm is more context-dependent and can deal with multiple sources involving conflict more adequately than the current approaches available.

This paper is organized as follows. In Section 2, we briefly review possibilistic logic and its combination modes. In Section 3, we define some quality measures of possibilistic knowledge bases and the relaxation of the conjunctive merge. In Section 4, we first introduce the measures of conflict and agreement between two PKBs, we then describe how to generate largely partial maximal consistent subsets. In Section 5, the degrees of conflict and agreement between two PKBs are extended to multiple PKBs first, then a merging operators selection criterion is defined to facilitate the selection of a right merging operator for a right set of PKBs. Following this, an adaptive merging algorithm is proposed. In Section 6, we compare our algorithm with some adaptive merging rules proposed in the literature. Finally, we conclude the paper in Section 7.

2. Preliminaries

2.1. Possibilistic Logic

We consider a propositional language $\mathcal{L}_{PS}$ from a finite set PS of propositional symbols [DLP94]. The classical consequence relation is denoted as $\vdash$. An interpretation (or a possible world) is a function from PS to $\{0, 1\}$ and is a model of a formula ϕ iff $\omega(\phi) = 1$ where ω is an interpretation. We use Ω to represent the set of all possible interpretations and use $p, q, r,$ etc. to represent atoms in PS. We denote formulas in $\mathcal{L}_{PS}$ by $\phi, \psi, \gamma,$, etc..

In possibilistic logic, at the semantic level, the basic notion is a *possibility distribution*, denoted by π, which is a mapping from a set of interpretations Ω to the interval [0,1]. $\pi(\omega)$ represents the possibility degree of the interpretation ω with available beliefs. From a *possibility distribution* π, two measures defined on a set of propositional or first order formulas can be determined. One is the *possibility measure*, defined as $\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$, where $\models$ denotes the propositional logic satisfaction, i.e., ω is a model of ϕ. The other is the *necessity measure*, and is defined as $N(\phi) = 1 - \Pi(\neg\phi)$. $\Pi(\phi)$ evaluates the maximum consistency level of ϕ with respect to beliefs encoded by π and $N(\phi)$ corresponds to the certainty degree of ϕ from available beliefs encoded by π.

At the syntactic level, a formula, called a *possibilistic formula*, is represented by a pair $(\phi,\ \alpha)$ where ϕ is a formula and $\alpha \in [0,1]$. Possibilistic formula $(\phi,\ \alpha)$ means that the necessity degree of ϕ is at least equal to α, i.e. $N(\phi) \geq \alpha$. Pieces of uncertain information from a source are represented by a *possibilistic knowledge base* (PKB) which is a finite collection of possibilistic formulas of the form $B = \{(\phi_i,\ \alpha_i) : i = 1, ..., n\}$. In this paper, we only consider PKBs where every formula ϕ is a classical propositional formula. The classical base associated with B is denoted as B^*, namely $B^* = \{\phi_i | (\phi_i,\ \alpha_i) \in B\}$. A PKB B is consistent if and only if its classical base B^* is consistent.

Given a PKB B, a unique *possibility distribution*, denoted as π_B, can be obtained by the principle of minimum specificity. For all $\omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall(\phi_i,\ \alpha_i) \in B, \omega \models \phi_i, \\ 1 - max\{\alpha_i | \omega \not\models \phi_i\} & \text{otherwise.} \end{cases} \tag{1}$$

The *inconsistency degree* of B, which defines the level of inconsistency of B, is defined by

$$Inc(B) = 1 - max_\omega \pi_{\mathcal{B}}(\omega).$$

Definition 2.1. [DLP94] Let B be a PKB, and $\alpha \in [0,1]$. We call the α-cut (respectively strict α-cut) of B, denoted as $B_{\geq\alpha}$ (respectively $B_{>\alpha}$), the set of classical formulas in B having a necessity degree at least equal to α (respectively strictly greater than α).

The inconsistency degree of B in terms of the α-cut can be equivalently defined as [DLP94]:

$$Inc(B) = max\{\alpha_i | B_{\geq\alpha_i}\ is\ inconsistent\}.$$

Definition 2.2. [DLP94] Let B be a PKB. Let $(\phi,\ \alpha)$ be a piece of information with $\alpha > Inc(B)$. $(\phi,\ \alpha)$ is said to be a consequence of B, denoted by $B \vdash_\pi (\phi,\ \alpha)$, iff $B_{\geq\alpha} \vdash \phi$. Given two PKBs B_1 and B_2, $B_1 \vdash_\pi B_2$ iff $B_1 \vdash (\phi, \alpha)$ for all $(\phi, \alpha) \in B_2$. We say B_1 is equivalent to B_2 iff $B_1 \vdash_\pi B_2$ and $B_2 \vdash_\pi B_1$.

2.2. Merging operators in possibilistic logic

Many combination operators for merging PKBs have been proposed [BDKP02]. Given several PKBs, the semantic combination rules are applied to aggregate the possibility distributions associated with them.

Definition 2.3. [BDKP02] A conjunctive operator is a two place function $\oplus : [0,1] \times [0,1] \rightarrow [0,1]$ such that $\forall \alpha \in [0,1]$, $\alpha \oplus 1 = 1 \oplus \alpha = \alpha$.

Definition 2.4. [BDKP02] A disjunctive operator is a two place function $\oplus : [0,1] \times [0,1] \rightarrow [0,1]$ such that $\forall \alpha \in [0,1]$, $\alpha \oplus 0 = 0 \oplus \alpha = \alpha$.

Examples of conjunctive operators are the *minimum* operator (also called the idempotent conjunction), the *product* ($\alpha \times \beta$) and the *linear product* ($max(0, \alpha + \beta - 1)$), and examples of disjunctive operators are the *maximum* operator (also called the idempotent disjunction), the *probabilistic sum* ($\alpha + \beta - \alpha \times \beta$) and the *bounded sum* ($min(1, \alpha + \beta)$).

Definition 2.5. [BDKP02] A reinforcement operator is a two place function $\oplus : [0,1] \times [0,1] \rightarrow [0,1]$ such that $\forall \alpha, \beta \neq 1$ and $\alpha, \beta \neq 0$, $\alpha \oplus \beta < min(\alpha, \beta)$.

Examples of reinforcement operators are the product and the linear product (also called *Lukasiewicz t-norm*). It is clear that a conjunctive operator can also be a reinforcement operator.

Given two PKBs B_1 and B_2, and a merging operator $\oplus$, the semantic combination rule combines the possibility distributions π_{B_1} and π_{B_2} using $\oplus$ as $\pi_{\oplus}(w) = \pi_{B_1}(w) \oplus \pi_{B_2}(w)$. Its syntactical counterpart is the following PKB [BDKP02]:

$$B_1 \oplus B_2 = \{(\phi_i, 1-(1-\alpha_i)\oplus 1) : (\phi_i, \alpha_i) \in B_1\} \cup \{(\psi_j, 1-1\oplus(1-\beta_j)) : (\psi_j, \beta_j) \in B_2\} \cup \{(\phi_i \vee \psi_j, 1-(1-\alpha_i)\oplus(1-\beta_j)) : (\phi_i, \alpha_i) \in B_1 \; and \; (\psi_j, \beta_j) \in B_2\}. \quad (2)$$

For example, when $\oplus = min$, $B_1 \oplus B_2 = B_1 \cup B_2$. It is often assumed that an operator used to combine possibility distributions should be both commutative and associative. In this case, the order of the combination will not influence the result of merging when multiple PKBs are considered. The general convention of selecting a merging operator in possibilistic logic is that when the union of original PKBs is consistent, it is advisable to use either a conjunctive or a reinforcement operator based combination rule because all the formulas in these PKBs are kept in the resulting PKB and their necessity degrees are not decreased; when the original knowledge bases are in conflict, a disjunctive operator is more appropriate.

It is clear that when $\oplus$ is associative, the syntactic computation of the resulting base is easily generalized to n sources. The syntactical generalization for a non-associative operator can be done as follows.

Proposition 2.1. [BDKP02] Let $B_1, ..., B_n$ be a set of n possibilistic knowledge bases and $\pi_1, ..., \pi_n$ be their associated possibility distributions. Let $\pi_{B_\oplus}$ be the result of combining $\pi_1, ..., \pi_n$ with $\oplus$. The possibilistic knowledge base associated to $\pi_{B_\oplus}$ is:

$$B_\oplus = \{(D_j, 1 - \oplus(x_1, ..., x_n)) : j = 1, ..., n\}, \quad (3)$$

where D_j are disjunctions of size j between formulas taken from different B_i's ($i = 1, ..., n$) and x_i is either equal to $1 - \alpha_i$ or to 1 depending respectively if ϕ_i belongs to D_j or not.

3. Quality Measures of Possibilistic Knowledge Bases

3.1. Quality measures of possibilistic knowledge bases

Measure of non-specificity: In [HK83], a measure of possibilistic uncertainty, called *nonspecificity*, was proposed to generalize the Hartley measure of information [Har28]. Given a possibility distribution π on $\Omega = \{\omega_1, ...\omega_n\}$, $\pi(\omega_i)$ ($i = 1, ..., n$) are reordered as $\pi_1 = l \geq \pi_2 \geq ... \geq \pi_n > 0$, where l may be less than 1 (in that case, π is not normal). Let $\pi_{n+1} = 0$. The measure of non-specificity of π is

$$H(\pi) = \frac{1}{l} \Sigma_{j=1}^{n} (\pi_j - \pi_{j+1}) log_2 j \quad (4)$$

Given two PKBs B_1 and B_2, we say the quality of B_1 is better than that of B_2 if $H(\pi_{B_1}) < H(\pi_{B_2})$, where π_{B_i} are possibility distributions of B_i.

Note that the definition of $H(\pi)$ given in Equation 4 is valid only when $min_{\omega \in \Omega} \pi(\omega) = 0$. When this condition does not hold, we need to add an extra element ω' into Ω and let $\pi(\omega') = 0$ in order to use Equation 4. It should also be noted that $H(\pi) = 0$ whenever $\pi(\omega_1) = l$ and $\pi(\omega_i) = 0$ for any $\omega_i \neq \omega_1$, regardless the actual value of l. This raises a question as whether $\pi(\omega_1) = 1$ should be treated the same as $\pi'(\omega_1) = 0.8$ (or any other values, such as 0.2). We argue on the one hand that when $\pi(\omega_1)$ is very small, e.g., 0.2, it is unlikely that $\pi(\omega_i) = 0$ holds for all the other ω_i, so most of the time, $\pi(\omega_1)$ cannot be too small. On the other hand, if $\pi(\omega_1)$ is the only non-zero value and it is reasonably large, then this possibility distribution is specific enough, since degrees of possibility can be viewed as relative measures after all. Therefore, $\pi(\omega_1) = 0.8$ can be regarded as carrying the same information as $\pi(\omega_1) = 1$ when $\pi(\omega_i) = 0$ for any other ω_i.

Degree of coherence: The degree of coherence was proposed to measure consistency in an inconsistent possibilistic knowledge base [DKP03]. It is defined in the framework of quasi-possibilistic logic.

First we introduce the quasi-possibilistic interpretation.

Definition 3.1. [DKP03] Let $\mathcal{O}_{PS}$ be the set defined as follows:

$$\mathcal{O}_{PS} = \{(+p, \alpha) : p{\in}PS, \alpha{\in}[0, 1]\} \cup \{(-p, \alpha) : p{\in}PS, \alpha{\in}[0, 1]\}$$

We call any $X{\subseteq}\mathcal{O}_{PS}$ a quasi-possibilistic interpretation. By Definition 3.1, there may exist several $(+p, \alpha_1)$,...,$(+p, \alpha_n)$ in an interpretation. In this case, we assume that only $(+p, \alpha_k)$ where $\alpha_k = max_{i=1}^{n}\alpha_i$ is taken into account and all the other formulas $(+p, \alpha_j)$ ($\alpha_j{\neq}\alpha_k$) do not appear in the interpretation (we can do so because any $(+p, \alpha_j)$ with $\alpha_j < \alpha_k$ is subsumed by $(+p, \alpha_k)$).

For each atom $p{\in}PS$, and each $X{\subseteq}\mathcal{O}_{PS}$, $(+p, \alpha){\in}X$ means that X provides a reason for p with confidence α and a reason against $\neg p$ with confidence α. Similarly, $(-p, \alpha){\in}X$ means that X provides a reason for $\neg p$ with confidence α and a reason against p with confidence α.

For an interpretation X, it may contain both $(+p, \alpha)$ and $(-p, \beta)$ for some atom p.

Definition 3.2. [DKP03] Let $l_1{\vee}...{\vee}l_n$ be a clause, then $Focus(l_1{\vee}...{\vee}l_n, l_i)$ is the clause without the disjunct l_i, i.e. $Focus(l_1{\vee}...{\vee}l_n, l_i) = l_1{\vee}...{\vee}l_{i-1}{\vee}l_{i+1}{\vee}...{\vee}l_n$.

Definition 3.3. [DKP03] Let p be a propositional symbol, $\sim$ is the complementary operation defined as ${\sim}p$ is $\neg p$ and ${\sim}(\neg p)$ is p. This operation is not in the object language but will be used to make definitions clearer.

Definition 3.4. [DKP03] For a model (interpretation) X, the strong satisfaction relation $\models_S$ is defined as follows. Let p be a propositional symbol, let $l_1, ..., l_n$ be literals, and let ϕ and ψ be two formulas:

- $X{\models_S}(p, \alpha)$ iff $(+p, \beta){\in}X$ with $\beta{\geq}\alpha$
- $X{\models_S}(\neg p, \alpha)$ iff $(-p, \beta){\in}X$ with $\beta{\geq}\alpha$
- $X{\models_S}(\phi{\wedge}\psi, \alpha)$ iff $X{\models_S}(\phi, \alpha)$ and $X{\models_S}(\psi, \alpha)$
- $X{\models_S}(l_1{\vee}...{\vee}l_n, \alpha)$ iff $(X{\models_S}(l_1, \alpha)$ or ... or $X{\models_S}(l_n, \alpha))$ and $\forall i{\in}\{1, ..., n\}$ (if $X{\models_S}({\sim}l_i, \alpha)$, then $X{\models_S}(Focus(l_1{\vee}...{\vee}l_n, l_i), \alpha)$)

Let us denote $Q\Pi(B)$ the set of strong models of B.

Example 3.1. Let $B = \{(p, 0.8), (\neg p{\wedge}q, 0.5), (\neg p{\vee}r, 0.2), (t{\vee}s, 0.3)\}$. Then $X = \{(+p, 0.8), (-p, 0.5), (+q, 0.5), (+r, 0.2), (t, 0.3)\}$ and $Y = \{(+p, 0.8), (-p, 0.5), (+q, 0.5), (+r, 0.2), (s, 0.3)\}$ are two strong models of K.

Next we introduce the notion of minimal quasi-possibilistic model.

Given a model X, we define the set of models that *subsum* it as follows:

$$subsum(X) = \{Y | Y{\neq}X, Y^*{\subseteq}X^* \; and \; \forall(+p_i,\alpha){\in}X, \exists(+p_i,\beta){\in}Y \; with \; \beta{\leq}\alpha, \\ and \; \forall(-p_i,\alpha){\in}X, \exists(-p_i,\beta){\in}Y \; with \; \beta{\leq}\alpha\}$$

Definition 3.5. [DKP03] The set of minimal (strong) models of B is defined as:

$$MQ\Pi(B) = \{X{\in}Q\Pi(B) | \forall Y{\in}Q\Pi(B), \; Y{\notin}subsum(X)\}$$

Before introducing the degree of coherence, we need to define the notions of $Conflictbase_{Q\Pi}$ and of $Opinionbase_{Q\Pi}$. We use $\pm p$ as a notion for $+p$ or $-p$.

Definition 3.6. [DKP03] Let X be a model,
$Confictbase_{Q\Pi}(X) = \{(p,\alpha) | (+p,\beta){\in}X \; and \; (-p,\gamma) \in X \; and \; \alpha = min(\beta,\gamma)\}$
$Opinionbase_{Q\Pi}(X) = \{(p,\alpha) | (\pm p,\alpha){\in}X \; and \; \nexists(\pm p,\beta) \in X \; with \; \beta{>}\alpha\}$

Definition 3.7. [DKP03] Let B be a set of pair (p,α), where $p{\in}PS$, then $\mathcal{A}(B) = \Sigma_{(p,\alpha)\in B}\alpha$.

The degree of coherence of a model and the degree of coherence of a knowledge base are then defined as

Definition 3.8. [DKP03] Let X be a model, then $Coherence_{Q\Pi}$ is a function from the set of interpretations to [0,1] such that

$$Coherence_{Q\Pi}(X) = 1 - \frac{\mathcal{A}(Conflictbase_{Q\Pi}(X))}{\mathcal{A}(Opinionbase_{Q\Pi}(X))}$$

Definition 3.9. [DKP03] Let B be a possibilistic knowledge base, then $Coherence_{Q\Pi}(B)$ is defined as:

$$Coherence_{Q\Pi}(B) = max_{X\in MQ\Pi}Coherence_{Q\Pi}(X)$$

For instance, given the knowledge base B and the two strong models X and Y in Example 3.1, the degrees of coherence for these two models respectively are $Coherence_{Q\Pi}(X) = 0.72$, $Coherence_{Q\Pi}(Y) = 0.72$. Since these two strong models are also minimal, we have $Coherence_{Q\Pi}(B) = 0.72$.

Quality measure of conjunctive and disjunctive merges When a disjunctive operator (e.g. the maximum) is applied during a merge, the merged information gets less precise. In contrast, if a conjunctive operator (e.g. the minimum) is applied, the resulting PKB may be inconsistent. Therefore, the associated possibility distribution π should be normalized first. The denominator l in Equation 4 normalizes such a distribution when measuring it. To facilitate the selection of conjunctive and disjunctive operators, we measure the difference of the non-specificities of a conjunctively merged PKB (denoted as B_{cm}) and a disjunctively merged PKB (denoted as B_{dm}). The *difference of the non-specificities* between B_{cm} and B_{dm} is defined as $\mathsf{Diff}_{B_{dm}}^{B_{cm}} = H(\pi_{B_{cm}}) - H(\pi_{B_{dm}})$.

When $\mathsf{Diff}_{B_{dm}}^{B_{cm}} < 0$, the quality of the conjunctive merge is better than the quality of the disjunctive merge. The smaller the $\mathsf{Diff}_{B_{dm}}^{B_{cm}}$ is the better the quality of the conjunctive merge, and it is more advisable to choose a conjunctive operator even though the resulting PKB may be inconsistent; otherwise, the disjunctive merge is a better choice.

Example 3.2. Suppose we are given three PKBs, $\mathcal{B} = \{B_1, B_2, B_3\}$: $B_1 = \{(p, 0.5), (q, 0.6), (r, 0.8)\}$, $B_2 = \{(p, 0.6), (q \vee r, 0.7)\}$, $B_3 = \{(\neg p \vee r, 0.4), (\neg q, 0.3)\}$. $B_{cm} = \{(p, 0.6), (q, 0.6), (r, 0.8), (\neg q, 0.3)\}$ and $B_{dm} = \{(\neg p \vee q \vee r, 0.4), (p \vee \neg q, 0.3)\}$. So $H(\pi_{B_{cm}}) = 1.44$ and $H(\pi_{B_{dm}}) = 2.76$, and $\mathsf{Diff}^{B_{cm}}_{B_{dm}} = -1.32$. Therefore, the quality of the minimum merge is better than that of the maximum merge.

3.2. Relaxation of the conjunctive rule

When a disjunctive operator results in a great loss of original information, a conjunctive operator may be more appropriate even though the result could be inconsistent. In the following we define some conditions where a conjunctive operator is advisable to be used.

Definition 3.10. Let $\mathcal{B} = \{B_1, ..., B_n\}$ be a set of n PKBs and let B_{cm} and B_{dm} be the merged PKBs of applying the minimum and maximum operators to $\mathcal{B}$ respectively. Then these PKBs in $\mathcal{B}$

(a) **should be merged conjunctively** when $Inc(B_{cm}) = 0$;

(b) **are advised to be merged conjunctively** when

(i) $0 < Inc(B_{cm}) \leq \epsilon_0$;

(ii) $Coherence_{Q\Pi}(B_{cm}) \geq \epsilon_1$;

(iii) $\mathsf{Diff}^{B_{cm}}_{B_{dm}} < 0$

(c) **should be merged disjunctively**, otherwise.

where ϵ_0 is a predefined threshold for the degree of inconsistency tolerance, and ϵ_1 is a threshold for the degree of coherence.

In Definition 3.10, we relax the condition where a conjunctive operator can be applied. For a set of PKBs which is inconsistent (that is, their union is inconsistent), when the degree of inconsistency and the degree of coherence of their conjunctively merged PKB are tolerable, and the quality of their disjunctively merged PKB is poorer than that of the conjunctively merged PKB, it is more appropriate to have them merged conjunctively. It is also possible to define another threshold for $\mathsf{Diff}^{B_{cm}}_{B_{dm}}$ which guarantees that the conjunctively merged result is *much* better than the disjunctively merged one.

Note that the minimum and the maximum operators are only used in Definition 3.10 to judge whether we should combine PKBs in $\mathcal{B}$ using a conjunctive or a disjunctive operator. Since the minimum operator has no reinforcement effect whilst the other two conjunctive operators have, it is sufficient to use minimum operator in Definition 3.10. The reason for this is that if these PKBs cannot be merged by the minimum operator, then they definitely cannot be merged by the other two conjunctive operators.

Furthermore, as we will see later, in our algorithm, when a set of PKBs can be merged conjunctively or are advised to be merged conjunctively, we are not restricted to use the minimum operator to combine them. We may choose a conjunctive operator with the reinforcement effect.

Example 3.3. (Continuing Example 3.2) Let us set $\epsilon_0 = 0.3$ and $\epsilon_1 = 0.6$ in Definition 3.10, which means that the requirement for the degree of inconsistency tolerance is low and the requirement for the degree of coherence is somewhat high. Since $Inc(B_1 \cup B_2 \cup B_3) = 0.3$, $Coherence_{Q\Pi}(B_1 \cup B_2$

$\cup B_3) = 0.85$ and $\text{Diff}^{B_{cm}}_{B_{dm}} = -1.32$, it is advisable to merge them conjunctively. Suppose we choose the minimum operator to merge them, then the result of merging is $B = B_1 \cup B_2 \cup B_3$. It is easy to check that $B_{cm} \vdash_\pi B_{dm}$ where B_{cm} and B_{dm} are given in Example 3.2. So B_{cm} contains more information than B_{dm} does.

4. Largely Partially Maximal Consistent Subset

4.1. Degree of conflict and degree of strong agreement

In this section, we introduce the degree of conflict and degree of agreement between two PKBs in [QLB05] that will be used in the next subsection and will be used to define the degree of conflict and agreement among multiple PKBs later.

Let us first define weighted prime implicants which generalize the prime implicants for PKB $B = \{(\phi_1, \alpha_1), ..., (\phi_n, \alpha_n)\}$ where ϕ_i are clauses and each of the clauses is a disjunction of literals. For a more general PKB, we can decompose it as an equivalent PKB whose formulas are clauses by the min-decomposability of necessity measures, i.e., $N(\wedge_{i=1,k}\phi_i) \geq m \Leftrightarrow \forall i, N(\phi_i) \geq m$ [DKP03].

Let $B = \{(\phi_1, \alpha_1), ..., (\phi_n, \alpha_n)\}$ be a PKB where ϕ_i are clauses. A weighted implicant of B is $D = \{(\psi_1, \beta_1), ..., (\psi_k, \beta_k)\}$ (which is also a PKB), such that $D \vdash_\pi B$, where ψ_i are literals such that no two complementary literals exist. Let D and D' be two weighted implicants of B, D is said to be *subsumed* by D' iff $D \neq D'$, $D'^* \subseteq D^*$ and $\forall (\psi_i, \alpha_i) \in D, \exists (\psi_i, \beta_i) \in D'$ with $\beta_i \leq \alpha_i$ (β_i is 0 if $\psi_i \in D^*$ but $\psi_i \notin D'^*$).

Definition 4.1. Let $B = \{(\phi_1, \alpha_1), ..., (\phi_n, \alpha_n)\}$ be a PKB where ϕ_i are clauses. A weighted prime implicant (**WPI**) of B is D such that

1. D is a weighted implicant of B
2. $\nexists$ D' of B such that D is subsumed by D'.

Let us look at an example to illustrate how to construct WPIs.

Example 4.1. Let $B = \{(p, 0.8), (q \vee r, 0.5), (q \vee \neg s, 0.6)\}$ be a PKB. The WPIs of B are $D_1 = \{(p, 0.8), (q, 0.6)\}$, $D_2 = \{(p, 0.8), (r, 0.5), (\neg s, 0.6)\}$, and $D_3 = \{(p, 0.8), (q, 0.5), (\neg s, 0.6)\}$.

Definition 4.2. Let B_1 and B_2 be two PKBs, and C and D be WPIs of B_1 and B_2 respectively, then the quantity of conflict between C and D is defined as

$$q_{Con}(C, D) = \sum_{(l,\alpha) \in C \text{ and } (\sim l, \beta) \in D} min(\alpha, \beta). \quad (5)$$

the quantity of strong agreement between C and D is defined as

$$q_{SA}(C, D) = \sum_{(l,\alpha) \in C,\ (l,\beta) \in D} min(\alpha, \beta), \quad (6)$$

and the quantity of weak agreement between C and D is defined as

$$q_{WA}(C,D) = \sum_{(l_i,\alpha_i)\in C\cup D, l_i\notin C^*\cap D^* \ and \ \sim l_i\notin C^*\cup D^*} \alpha_i. \quad (7)$$

When the weights associated with all the formulas are 1, $q_{Con}(C,D)$ is the cardinality of the set of literals which are in conflict in $C\cup D$; $q_{SA}(C,D)$ is the cardinality of the set of literals that are in both C and D; $q_{WA}(C,D)$ is the cardinality of the set of literals which are in either C or D but not both.

We now define the degree of conflict.

Definition 4.3. Let B_1 and B_2 be two PKBs. Let C and D be WPIs of B_1 and B_2 respectively. Let $Atom_C(C,D)$ denote the cardinality of the set of atoms which are in conflict in $C\cup D$, then the degree of conflict between C and D is defined as

$$d_{Con}(C,D) = \frac{q_{Con}(C,D)}{Atom_C(C,D) + q_{SA}(C,D) + \lambda q_{WA}(C,D)} \quad (8)$$

Let $Atom_{SA}(C,D)$ denote the cardinality of the set of atoms which are included in both C and D, then the degree of strong agreement between C and D is defined as

$$d_{SA}(C,D) = \frac{q_{SA}(C,D)}{Atom_{SA}(C,D) + q_{Con}(C,D) + \lambda q_{WA}(C,D)} \quad (9)$$

where $\lambda \in (0,1]$ is used to weaken the influence of the quantity of weak agreement on the degree of conflict and on the degree of strong agreement. In the following, we always assume that $\lambda = 0.5$, that is, the quantity of weak agreement only has "half" as much the influence on the degree of conflict (or on the degree of strong agreement) as the quantity of strong agreement.

Definition 4.4. Let B_1 and B_2 be two PKBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of WPIs of B_1 and B_2 respectively, then the degree of conflict between B_1 and B_2 is defined as

$$D_{Con}(B_1,B_2) = min\{d_{Con}(C,D) | C\in\mathcal{C},\ D\in\mathcal{D}\}, \quad (10)$$

and the degree of strong agreement between B_1 and B_2 is defined as

$$D_{SA}(B_1,B_2) = max\{d_{SA}(C,D) | C\in\mathcal{C},\ D\in\mathcal{D}\}. \quad (11)$$

Example 4.2. Let $B_1 = \{(p,0.8), (q\vee r,0.4), (p\rightarrow s,0.5)\}$ and $B_2 = \{(p\vee\neg r,0.8), (q,0.6), (\neg s,0.7)\}$. The WPIs of B_1 are $C_1 = \{(p,0.8), (q,0.4), (s,0.5)\}$ and $C_2 = \{(p,0.8), (r,0.4), (s,0.5)\}$, and the WPIs of B_2 are $D_1 = \{(p,0.8), (q,0.6), (\neg s,0.7)\}$ and $D_2 = \{(\neg r,0.8), (q,0.6), (\neg s,0.7)\}$. So $d_{Con}(C_1,D_1) = 0.22$, $d_{Con}(C_1,D_2) = 0.22$, $d_{Con}(C_2,D_1) = 0.217$, $d_{Con}(C_2,D_2) = 0.33$. Therefore, $D_{Con}(B_1,B_2) = 0.217$. Furthermore, $d_{SA}(C_1,D_1) = 0.48$, $d_{SA}(C_1,D_2) = 0.17$, $d_{SA}(C_2,D_1) = 0.4$, $d_{SA}(C_2,D_2) = 0$. Therefore, $D_{SA}(B_1,B_2) = 0.48$.

4.2. Ordering knowledge bases

Given a PKB B as a background knowledge base which is called a reference PKB, we can define an ordering relation between two PKBs in relation to reference B.

Definition 4.5. Let B, B_1 and B_2 be three PKBs which are self-consistent. A binary distance relation between B_1 and B_2 with reference B, denoted as $\preceq_B$, is defined as $B_1 \preceq_B B_2$ when one of the following conditions holds:

(a) $B \cup B_i$ ($i = 1, 2$) are consistent and $D_{SA}(B, B_1) \geq D_{SA}(B, B_2)$;

(b) $Inc(B \cup B_1) \leq Inc(B \cup B_2)$;

(c) $Inc(B \cup B_1) = Inc(B \cup B_2)$ and $D_{Con}(B, B_1) \leq D_{Con}(B, B_2)$.

As usual, we use $B_1 \prec_B B_2$ to denote $B_1 \preceq_B B_2$ but $B_2 \not\preceq_B B_1$, and we use $B_1 \sim_B B_2$ to denote $B_1 \preceq_B B_2$ and $B_2 \preceq_B B_1$. $B_1 \preceq_B B_2$ means that the distance between B and B_1 is not greater than that between B and B_2, so B_1 is more close to B than B_2 is.

4.3. Generating largely partially maximal consistent subsets (LPMCSs)

Definition 4.6. Let $\mathcal{B} = \{B_1, ..., B_n\}$ be a set of PKBs. A subset $S_\mathcal{B} \subseteq \mathcal{B}$ is called a LPMCS of $\mathcal{B}$ if the PKBs in $S_\mathcal{B}$ can be merged conjunctively or advised to be merged conjunctively, but PKBs in $S_\mathcal{B} \cup \{B_i\}$ cannot, where $B_i \in (\mathcal{B} \setminus S_\mathcal{B})$.

Example 4.3. (Continuing Example 3.3) Suppose a fourth PKB $B_4 = \{(\neg p, 0.6), (\neg q, 0.6)\}$ is given in addition to the three PKBs in Example 3.3. Then $S^1_\mathcal{B} = \{B_1, B_2, B_3\}$ and $S^2_\mathcal{B} = \{B_4\}$ are two LPMCSs.

Definition 4.7. Let $B, B_1, ..., B_n$ be $n + 1$ PKBs which are self-consistent and B be the reference PKB. The *preferred sequence of merging* with reference B is defined as $(B, B_{i_1}, ..., B_{i_j}, B_{i_{j+1}}, ..., B_{i_n})$ such that for any $1 \leq j < n$, $B_{i_j} \preceq_{B^{j-1}} B_{i_t}$ when $j < t \leq n$, where B^{j-1} is the union of first j PKBs in the sequence, and when $j = 1$, $B^0 = B$. Then $S_B = \{B, B_{i_1}, ..., B_{i_j}\}$ is the LPMCS with reference B such that the PKBs in S_B can be merged conjunctively or advised to be merged conjunctively, but PKBs in $S_B \cup \{B_{i_t}\}$ cannot, where $j < t \leq n$.

In Definition 4.7, we order the PKBs in a way such that if B_i is closer than B_{i+1} is to the unions of KPBs that are already ordered, then B_i is ordered in front of B_{i+1}. A reference PKB is chosen according to the non-specificity of each knowledge base. Usually, we start with a PKB that is more specific than other PKBs. If there are several candidates, we choose the one that is provided by the most reliable source.

Example 4.4. (Continuing Example 4.3) In Example 4.3, B_1 has the least non-specificity, so it is chosen as the reference. By Definition 4.7, the preferred sequence of merging with reference B_1 is (B_1, B_2, B_3, B_4) and the corresponding LPMCS is $S_\mathcal{B} = \{B_1, B_2, B_3\}$, since we have $Inc(B_1 \cup B_2 \cup B_3) = 0.3$, $Coherence_{Q\Pi}\ (B_1 \cup B_2 \cup B_3) = 0.85$ and $\mathrm{Diff}^{B_{cm}}_{B_{dm}} = -1.32$, whilst $Inc(B_1 \cup ..., \cup B_4) = 0.6 > 0.3$.

In this example, if we set $\epsilon_0 = 0$ in Definition 3.10 and we still choose B_1 as the reference, then the two LPMCSs are $S^1_\mathcal{B} = \{B_1, B_2\}$ and $S^2_\mathcal{B} = \{B_3, B_4\}$ which are both in fact maximal consistent subsets.

5. Context Dependent Merging

In this section, we propose an adaptive merging algorithm which deals with multiple LPMCSs and their maximal consistent subsets using different merging operators.

5.1. Degree of strong agreement among multiple knowledge bases

We extend the definition of the degree of strong agreement between two WPIs to multiple WPIs in order to define the degree of strong agreement among multiple PKBs.

Definition 5.1. Let $\mathcal{B} = \{B_1, ..., B_n\}$ be a set of n PKBs such that $\cup_{i=1}^{n} B_i$ is consistent. Let C_i be a WPI of B_i ($i = 1, ..., n$), and $\mathcal{C} = \{C_i : i = 1, ..., n\}$. Let $Atom_{SA}(\mathcal{C}) = | \cap_i C_i^*|$, $q_{SA}(\mathcal{C}) = \Sigma_{(l,a_1)\in C_1 \& (l,a_2)\in C_2 \& ... \& (l,a_n)\in C_n, \forall i, a_i \neq 0} min(a_i)$ and $q_{WA}(\mathcal{C}) = \Sigma_{(l,a_i)\in C_i, \exists j, a_j = 0} min\{a_i : a_i \neq 0\}$. Let $q_{Con}(\mathcal{C}) = \Sigma_{l, \exists i,j,\ such\ that\ (l,a_i)\in C_i\ and\ (\sim l, a_j)\in C_j} min(a_i : (\phi_i, a_i) \in C_i, \phi_i = l\ or\ \sim l\ )$. Then the degree of strong agreement among C_i is defined as

$$d_{SA}(\mathcal{C}) = \frac{q_{SA}(\mathcal{C})}{Atom_{SA}(\mathcal{C}) + q_{Con}(\mathcal{C}) + \lambda q_{WA}(\mathcal{C})}, \tag{12}$$

where $\lambda \in (0, 1]$ is used to weaken the influence of the quantity of weak agreement on the degree of strong agreement. As usual, we take $\lambda = 0.5$.

It is easy to check that we have the following result.

Proposition 5.1. Let B_1, B_2 be two PKBs. Let C_1, C_2 be WPIs of B_1, B_2 respectively, and $\mathcal{C} = \{C_1, C_2\}$. Let $d_{SA}(C_1, C_2)$ and $d_{SA}(\mathcal{C})$ be the degrees of strong agreement obtained by Equation 9 and Equation 12 respectively. Then $d_{SA}(C_1, C_2) = d_{SA}(\mathcal{C})$.

Definition 5.2. Let $\mathcal{B} = \{B_1, ..., B_n\}$ be a set of n PKBs such that $\cup_{i=1}^{n} B_i$ is consistent. Let $\mathcal{C}_i$ be the set of WPIs of B_i respectively. Then the degree of strong agreement among B_i is defined as

$$D_{SA}(\mathcal{B}) = max\{d_{SA}(C_{1,j_1}, ..., C_{n,j_n}) | C_{1,j_1} \in \mathcal{C}_1, ... C_{n,j_n} \in \mathcal{C}_n\} \tag{13}$$

It is clear that Definition 5.2 generalizes Definition 4.4 when the set of knowledge bases are consistent.

Example 5.1. Suppose we are given a set of three PKBs: $B_1 = \{(p, 0.5), (q, 0.6), (r, 0.8)\}$, $B_2 = \{(p, 0.6), (q \vee r, 0.7)\}$, $B_3 = \{(p, 0.8), (\neg p \vee r, 0.4)\}$. The WPI for B_1 is $C_1 = \{(p, 0.5), (q, 0.6), (r, 0.8)\}$. The WPIs for B_2 are $D_1 = \{(p, 0.6), (q, 0.7)\}$ and $D_2 = \{(p, 0.6), (r, 0.7)\}$. The WPIs for B_3 are $S_1 = \{(p, 0.8), (\neg p, 0.4)\}$ and $S_2 = \{(p, 0.8), (r, 0.4)\}$. Therefore, $d_{SA}(\{C_1, D_1, S_1\}) = 0.23$, $d_{SA}(\{C_1, D_1, S_2\}) = 0.33$, $d_{SA}(\{C_1, D_2, S_1\}) = 0.22$, and $d_{SA}(\{C_1, D_2, S_2\}) = 0.39$, as a consequence, $D_{SA}(B_1, B_2, B_3) = 0.39$.

5.2. Merging operators selection criteria

Given a set of consistent PKBs, if the degree of strong agreement between them is very high, then they share beliefs on most of the topics. In this case, it is advisable to combine them using an operator with higher *reinforcement* effect, for example, the linear product $max(0, \alpha + \beta - 1)$. However, if the degree of strong agreement among them is low, it is advisable to combine them using the minimum operator which does not have any *reinforcement* effect.

Definition 5.3. Let the merging operators be the minimum operator min, the product operator $\times$, and the linear product, then we have the following criteria to choose an operator.

If $D_{SA}(B_1, ..., B_n) \geq 0.8$, the merging operator is the linear product;
If $D_{SA}(B_1, ..., B_n) \in (0.3, 0.8)$, the merging operator is the product operator;
If $D_{SA}(B_1, ..., B_n \leq 0.3$, the merging operator is the minimum operator.

That is, we choose the linear product when the degree of strong agreement among B_i is high; we choose the minimum operator when the degree of strong agreement among them is low; and we choose the product operator otherwise.

When considering two sets of PKBs, we use the following criterion to decide which set of PKBs should use what operator.

Definition 5.4. Let $\mathcal{B}_1 = \{B_1^1, ..., B_n^1\}$, and $\mathcal{B}_2 = \{B_1^2, ..., B_m^2\}$ be two sets of PKBs where both $\cup_i B_i^1$ and $\cup_i B_i^2$ are consistent. When $D_{SA}(\mathcal{B}_1) < D_{SA}(\mathcal{B}_2)$, the operators $\oplus_1$ and $\oplus_2$ selected for merging PKBs in $\mathcal{B}_1$ and $\mathcal{B}_2$ should satisfy $\oplus_1(\alpha, \beta) \geq \oplus_2 (\alpha, \beta)$ for all α, $\beta \in [0, 1]$.

This definition says that an operator applied to a set of PKBs which are in strong agreement should have more reinforcement effect than an operator applied to a set of PKBs which are less agreeable with each other.

5.3. The algorithm

We now propose an adaptive algorithm to merge multiple PKBs. The basic idea of the algorithm is described as follows. Let $\mathcal{B} = \{B_1, ..., B_n\}$ be a knowledge profile, a set of PKBs. We first select a PKB B_i with the least non-specificity. This step is a competition step. The PKB which contains more information than other PKBs wins the game and is selected. In the second step, we generate a LPMCS with a chosen reference B_i. That is, we $w.r.t$ B_i find a group of PKBs such that these belief bases can be merged with B_i conjunctively or are advised to be merged with B_i conjunctively. We then find the unique maximal consistent subset of the LPMCS. The PKBs in this maximal consistent subset are merged by a chosen conjunctive operator based on their degree of strong agreement. The result of merging is then merged with other PKBs in the LPMCS using the minimum operator. Those PKBs in the PLMCS are then deleted from the knowledge profile $\mathcal{B}$ and the remaining PBKs in $\mathcal{B}$ are dealt with the above steps repeatedly, until $\mathcal{B}$ is empty.

Adaptive Algorithm: $\mathcal{B} = \{B_1, ..., B_n\}$ is a set of PKBs; $\epsilon_0, \epsilon_1 \in [0, 1]$, assume $\epsilon_0 = 0.3$ and $\epsilon_1 = 0.6$.
Begin
m=1.
while $|\mathcal{B}| > 0$ **do**

Step 1 Select B_i with the least non-specificity in $\mathcal{B}$ (choose the one with the best quality if there are several candidates);

Step 2 Generate the LPMCS with reference B_i $S_m = \{B_i, B_{i_1}, ..., B_{i_j}\}$ (PKBs in S_m are at least advised to be merged conjunctively);

Step 3 Find l ($l \leq j$) such that $B_i \cup B_{i_1} \cup ... \cup B_{i_l}$ is consistent and $B_i \cup B_{i_1} \cup ... \cup B_{i_l} \cup B_{i_{l+1}}$ is not;

Step 4 $\pi^m(\omega) = min\{\pi_i(\omega) \oplus \pi_{i_1}(\omega) \oplus ... \oplus \pi_{i_l}(\omega), \pi_{i_{l+1}}(\omega), ..., \pi_{i_j}(\omega)\}$, where $\oplus$ is the conjunctive operator chosen according to $D_{SA}(B_i, B_{i_1}, ..., B_{i_l})$ in Definition 5.3;

Let $\mathcal{B} = \mathcal{B} \backslash S_m$, $m = m + 1$.

End while

$\pi^n(\omega) = max\{\pi^1(\omega), ..., \pi^{m-1}(\omega)\}$.

$LPMCS(\mathcal{B}) = m$.

End

π^n is the possibility distribution of the final merge result of all the PKBs in $\mathcal{B}$.

The syntactical counterpart of the adaptive algorithm can be easily defined based on Equation 3. In Step 4, suppose $\oplus$ is the chosen conjunctive operator, then the syntactical counterpart of $\pi^m(\omega)$ is $B_{\oplus} \cup B_{i_{l+1}} \cup ... \cup B_{i_j}$, where $B_{\oplus}$ is the syntactically merged base of B_i, B_{i_1},..., and B_{i_l} by Equation 3. Suppose B^i ($i = 1, ..., m-1$) are the syntactical counterparts of π^i respectively, then the syntactical counterpart of π^n is $B^n = \{(\phi_1 \vee ... \vee \phi_{m-1}, min(\alpha_1, ..., \alpha_{m-1})) : (\phi_i, \alpha_i) \in B^i, i = 1, ..., m-1\}$.

It is clear that the algorithm is nondeterministic if we have several choices in Step 1, that is, the selection of reference PKB in Step 1 can influence subsequent PLMCSs. When several PKBs have the least non-specificity in this step, we need to make use of the knowledge of reliability of each source [DP94]. In this case, a source with a higher degree of reliability should be selected. Therefore, given a knowledge profile and necessary knowledge about the reliability of some sources, this merging algorithm should have a unique output.

When $LPMCS(\mathcal{B}) = 2$ and the conjunctive operator is minimum in Step 4, then $\pi^n(\omega) = min\{\pi_1(\omega), ..., \pi_n(\omega)\}$. When $LPMCS(\mathcal{B}) = n + 1$, $\pi^n(\omega) = max\{\pi_1(\omega), ..., \pi_n(\omega)\}$. Clearly, we have the following two propositions.

Proposition 5.2. Suppose $\oplus = min$ in Step 4 of the above algorithm, we have for each ω,

$$min(\pi_1(\omega), ..., \pi_n(\omega)) \leq \pi^n(\omega) \leq max(\pi_1(\omega), ..., \pi_n(\omega)),$$

where π_i are possibility distributions of B_i respectively.

Proposition 5.3. Let $\epsilon_0 = 0$ in Definition 3.10. Then $\pi^m(\omega) = \pi_i(\omega) \oplus \pi_{i_1}(\omega) \oplus ... \oplus \pi_{i_l}(\omega)$ in Step 4 of the algorithm, where $\oplus$ is the conjunctive operator chosen according to $D_{SA}(B_i, B_{i_1}, ..., B_{i_l})$.

Proposition 5.3 says that when the degree of inconsistency tolerance $\epsilon_0 = 0$, then each LPMCS in Step 2 is in fact a consistent subset, Step 3 is therefore redundant. When this happens and when operator $\oplus$ is defined as min, the algorithm is somehow equivalent to MCS-based approach discussed in Section 6. Now we use an example to illustrate the algorithm.

Example 5.2. (Continuing Example 4.4) By Example 4.4 we know that the largely partially maximal consistent subset is $\{B_1, B_2, B_3\}$. Since $B_1 \cup B_2$ is consistent and $B_1 \cup B_2 \cup B_3$ is not, we combine B_1 and B_2 by the product operator because $D_{SA}(B_1, B_2) = 0.48$. That is, the possibility distribution of the final merge result is $\pi(\omega) = max(min(\pi_1(\omega) \times \pi_2(\omega), \pi_3(\omega)), \pi_4(\omega))$. The PKB associated with π is $B = \{(\neg p \vee q, 0.6), (\neg p \vee r, 0.6), (p \vee \neg q, 0.6), (\neg q \vee r, 0.6), (\neg q, 0.3)\}$.

The output of the adaptive algorithm may be an inconsistent PKB. We have the following result for the inconsistency degree of the resulting PKB.

Proposition 5.4. Let ϵ_0 be the degree of inconsistency tolerance used in the adaptive algorithm. Suppose in each iteration of Step 4, the minimum is chosen. Let B be the output PKB of the *Adaptive Algorithm*. Then $Inc(B) \leq \epsilon_0$.

The computational complexity of the algorithm is analyzed as follows.

Proposition 5.5. The computational complexity of the algorithm is in Δ_2^p, where Δ_2^p is the class of all languages that can be recognized in polynomial time by a deterministic Turing machine equipped with an *NP* oracle.

Proposition 5.5 shows that the computation complexity of our algorithm is not much harder than propositional satisfiability.

6. Related Work

An adaptive rule in [DPT88] An adaptive rule was proposed which considered *j* sources out of all the sources, where it was assumed that these *j* sources are reliable. Since it was not known which *j* sources were reliable, all the subsets with cardinality *j* were considered. The intermediary conjunctively merged results are then merged disjunctively. Given a set of PKBs $\mathcal{B} = \{B_1, ..., B_N\}$ with π_i being the possibility distribution of B_i, the adaptive rule is defined as, for each $\omega \in \Omega$,

$$\pi_{(j)}(\omega) = max_{J \subseteq N, |J|=j}\{min_{i \in J}\{\pi_i(\omega)\}\} \tag{14}$$

A method to decide the value of *j* was given in [DP94]: let

$$m = max\{|T| : h(T) = 1\}, \;\; n = max\{|T| : h(T) > 0\},$$

$T \subseteq \mathcal{B}$ and $h(T) = max_\omega min_{B_i \in T} \pi_i(\omega)$, then, j is defined as m and N is defined as n in Equation 14, where n indicates that these n sources at least partially consistent and among them j sources are completely consistent.

The operator defined above suffers from several problems: First, once the value *j* is decided, all the subsets having *j* as the cardinality are selected for separate conjunctive merges. However, some selected subsets may contain sources which are in strong conflict and so it is not appropriate to merge them conjunctively; Second, if the value j is small, there may exists too many subsets with cardinality j. In this case, too much information will be lost after merging.

Example 6.1. (Continuing Example 4.4) It is easy to check that $m = 2$ and $n = 4$. Let $j = 2$ in Equation 14, then there are six subsets of $\mathcal{B}$ with cardinality j. Among them the subset $\{B_1, B_4\}$ contains two PKBs which are in strong conflict (their degree of conflict is 0.34 and their degree of inconsistency is 0.5). The result of merge is $B_{DP1} = \{(p \vee \neg q \vee r, 0.6), (\neg p \vee q \vee r, 0.6), (p \vee \neg q, 0.5), (\neg p \vee \neg q \vee r, 0.4), (\neg q \vee r, 0.3)\}$, which drops much more important information than the resulting PKBs using our algorithm.

Another adaptive rule in [DP92]: the rule proposed in [DP92] utilizes the maximum and the minimum operators. This operator is extended to more than two sources in [DP94] based on the adaptive rule in Equation 14. It is defined as follows:

$$\pi_{AD}(\omega) = max(\frac{\pi_{(n)}(\omega)}{h(n)}, min(\pi_{(m)}(\omega), 1 - h(n))), \tag{15}$$

where $h(n) = max\{h(T)||T| = n\}$ as defined previously and let $j = m$ and $N = n$ in Equation 14. This rule is more adaptive and context dependent than adaptive rule in Eq 14. However, it inherits the first problem of Equation 14.

Example 6.2. (Continuing Example 6.1) $h(\pi_{(n)}) = 0.4$. By Proposition 3 in [BDP97], the PKB associated with $\frac{\pi_{(n)}(s)}{h(n)}$ is $B' = \{(r, 0.5)\}$. We can also compute the PKB associated with $min(\pi_{(m)}(s), 1 - h(n))$ which is $B'' = \{(p \vee \neg q \vee r, 0.6), (\neg p \vee q \vee r, 0.6), (p \vee \neg q, 0.5), (\neg p \vee \neg q \vee r, 0.4)\}$. So the resulting PKB merged by Equation 15 is $B_{DP2} = \{(\neg p \vee q \vee r, 0.5), (p \vee \neg q \vee r, 0.5), (\neg p \vee \neg q \vee r, 0.4)\}$. Clearly, this PKB deletes too much original information as well.

MCS-based adaptive merging in [DFP00]: an adaptive operator based on maximal consistent subsets (MCS) of $\mathcal{B}$ was proposed in [DFP00]. Suppose $\mathcal{B}_1, ..., \mathcal{B}_k$ are all the maximal consistent subsets of $\mathcal{B}$, then the MCS-based operator is defined as

$$\pi_{MCS}(\omega) = max_{i=1,...,k} min_{B_j \in \mathcal{B}_i} \pi_{B_j}(\omega) \tag{16}$$

The MCS-based operator is more context dependent than the first two adaptive rules introduced in this section. However, it is computationally very difficult because the computation of maximal consistent subsets is Π_2^p-hard. This operator is appropriate when the number of maximal consistent subsets is not small and most original knowledge bases are not involved in conflict. In the case where most PKBs are involved in conflict, the result of merge using the MCS-based operator may delete too much information.

Example 6.3. (Continuing Example 4.4) There are three maximal consistent subsets: $\mathcal{B}_1 = \{B_1, B_2\}$, $\mathcal{B}_2 = \{B_2, B_3\}$ and $\mathcal{B}_3 = \{B_3, B_4\}$. So the resulting PKB of merging is $B_{MCS} = \{(p \vee \neg q, 0.6), (\neg p \vee q \vee r, 0.6), (\neg p \vee r, 0.4), (\neg p \vee \neg q \vee r, 0.4), (\neg p \vee \neg q \vee r, 0.3), (\neg q \vee r, 0.3)\}$. Let B be the resulting PKB of merging using our algorithm as shown in Example 5.2. It is easy to check that $B \vdash_\pi B_{MCS}$.

Split-combination approach in [QLG04]: we proposed a split-combination (S-C) merging method which also integrates both conjunctive and disjunctive operators in [QLG04]. This method consists of the following steps. Given a set of PKBs $\{B_1, ..., B_n\}$, let $B = B_1 \cup ... \cup B_n$, it first computes the *upper free degree* $Free_{upp}(B_1 \cup ... B_n)$ of the union of original PKBs, which is the minimum value such that formulas with weights greater than it are not involved in conflict in $B_1 \cup ... \cup B_n$. Then each PKB B_i is split into two subbases C_i and D_i, where $C_i = \{(\phi, \alpha) \in B_i : \alpha \leq Free_{upp}(B_1 \cup ... \cup B_n)\}$ and $D_i = B_i \backslash C_i$. After that, we combine all C_i by the maximum operator (or a disjunctive operator) into a PKB C and combine all D_i by the minimum operator (or a conjunctive operator) into a PKB D. Finally, the result of merging is $C \cup D$. When the upper free degree is low, the S-C method would be very desirable because it keeps most of the original information and weaken conflicting information. However, when the upper free degree is high, then most of the original information will be lost.

Table 1. Comparison of the merged results of the four methods discussed in this section and our approach for the four PKBs given in Example 4.4.

Methods	Merged PKB	Conclusion
Adaptive rule in [DPT88]	$B_{DP1} = \{(p \vee \neg q \vee r, 0.6), (\neg p \vee q \vee r, 0.6), (p \vee \neg q, 0.5), (\neg p \vee \neg q \vee r, 0.4), (\neg q \vee r, 0.3)\}$	$H(\pi_B) < H(\pi_{B_{DP1}})$
Adaptive rule in [DP92]	$B_{DP2} = \{(\neg p \vee q \vee r, 0.5), (p \vee \neg q \vee r, 0.5), (\neg p \vee \neg q \vee r, 0.4)\}$	$B \vdash_\pi B_{DP2}$
MCS-based merging	$B_{MCS} = \{(p \vee \neg q, 0.6), (\neg p \vee q \vee r, 0.6), (\neg p \vee r, 0.4), (\neg p \vee \neg q \vee r, 0.4), (\neg p \vee \neg q \vee r, 0.3), (\neg q \vee r, 0.3)\}$	$B \vdash_\pi B_{MCS}$
Split-combination method	$B_{S-C} = \{(p \vee \neg q, 0.3), (r, 0.8)\}$	$H(\pi_B) < H(\pi_{B_{S-C}})$
Our algorithm	$B = \{(\neg p \vee q, 0.6), (\neg p \vee r, 0.6), (p \vee \neg q, 0.6), (\neg q \vee r, 0.6), (\neg q, 0.3)\}$	

Example 6.4. (Continue Example 4.3) The upper free degree of $B_1 \cup ... \cup B_4$ is 0.6. So B_1 is split into $C_1 = \{(p, 0.5), (q, 0.6)\}$ and $D_1 = \{(r, 0.8)\}$, B_2 is split into $C_2 = \{(p, 0.6)\}$ and $D_2 = \{(q \vee r, 0.7)\}$, B_3 is split into $C_3 = \{(\neg p \vee r, 0.4), (\neg q, 0.3)\}$ and $D_3 = \emptyset$ and B_4 is split into $C_4 = \{(\neg p, 0.6), (\neg q, 0.6)\}$ and $D_4 = \emptyset$. C_i are combined by the maximum operator as $C = \{(p \vee \neg q, 0.3)\}$ and D_i are combined by the minimum operator as $D = \{(r, 0.8), (q \vee r, 0.7)\}$. So the result of merging is $B' = C \cup D = \{(p \vee \neg q, 0.3), (r, 0.8), (q \vee r, 0.7)\}$, which is equivalent to $\{(p \vee \neg q, 0.3), (r, 0.8)\}$. Compared with B in Example 5.2, B' is better in preserving formulas whose necessity degrees are greater than the upper free degree, for example, $(r, 0.8)$. However, B' loses too much information contained in formulas whose necessity degrees are under the upper free degree, that is, only $(p \vee \neg q, 0.3)$ is retained in B' after merging. In contrast, we have kept $(\neg p \vee q, 0.6)$, $(p \vee \neg q, 0.6)$ and $(\neg q, 0.3)$ in B.

The comparisons discussed above are summarized in Table 1 which shows that the merged result from our algorithm is better than that from the other approaches mentioned in this section.

7. Conclusions and Further Work

In this paper, we proposed an adaptive algorithm for merging n ($n > 2$) prioritized knowledge bases which extends the algorithm in [HL05] to the possibilistic logic framework. The idea is that when most of the sources are involved in conflict and it is not possible to get maximal consistent subsets that are sufficiently large, we then look for largely partially maximal consistent subsets such that each of them contains sources that are largely agreeable. These largely partially maximal consistent subsets of PKBs can be merged with the relaxation of the minimum operator after the maximal consistent subset in each of the LPMCSs is merged with a suitable conjunctive operator (maybe a reinforcement operator). In this way, we do not have to merge all the sources with a disjunctive operator, and therefore, the merged result should retain more information than a simple disjunctive merge. Compared with other merging methods, our method is more context dependent and may keep more important information when most PKBs are involved in conflict.

The idea of generating LPMCSs for adaptive merging was first proposed in [HL05] in the context of possibility theory where uncertain information was modelled with sets of weighted subsets. To apply this idea in possibilistic logic, we have incorporate the measures of degrees of conflict and agreements among multiple PKBs in [QLB05] and the measures of coherence of a merged PKB in [DKP03] in this paper.

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