Measuring Conflict and Agreement Between Two Prioritized Knowledge Bases in Possibilistic Logic[1]

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Abstract

In this paper we investigate the relationship between two prioritized knowledge bases by measuring both the conflict and the agreement between them. First of all, a quantity of conflict and two quantities of agreement are defined. The former is shown to be a generalization of the well-known *Dalal distance* which is the hamming distance between two interpretations. The latter are, respectively, a quantity of strong agreement which measures the amount of information on which two belief bases "totally" agree, and a quantity of weak agreement which measures the amount of information that is believed by one source but is unknown to the other. All three quantity measures are based on the *weighted prime implicant*, which represents beliefs in a prioritized belief base. We then define a degree of conflict and two degrees of agreement based on our quantity of conflict and quantities of agreement. We also consider the impact of these measures on belief merging and information source ordering.

Key words: Knowledge Representation and Reasoning, Possibilistic Logic, Conflict resolution, Automated Reasoning, Information Measures

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[1]This paper is extended and revised from a conference paper [28]

1. Introduction

Multiple agents need to interact and cooperate with each other to achieve the common goals or to resolve conflict. So it is important to know the relationship between knowledge bases of these agents, for example, whether their knowledge bases are in conflict with each other and to what extent their knowledge bases are in conflict. In the belief revision and belief merging literature, the well-known Dalal distance known as the Hamming distance between interpretations [11], plays a key role in the notion of minimal change. The Dalal distance between two interpretations models how many atoms are in conflict, so it measures only the quantity of conflict between them. Later, Hunter defined a degree of conflict between two knowledge bases based on the Dalal distance to define the believability of an argumentation tree [18].

In recent years, some measures of information and contradiction have been proposed. These measures can be used to define some ordering relations between two knowledge bases. In [23], a degree of contradiction and a degree of ignorance were defined and they can be used to order the sources of information. If a knowledge base has a high degree of contradiction and a low degree of ignorance, then it has a low order. In [16], a pre-order relation between two knowledge bases, called a compromise relation, was defined according to the quantities of conflicting information and total information in them.

In all the work described above, the quantity of conflicting information in a single knowledge base or between two knowledge bases was the main focus. However, in reality, when establishing the relationships between knowledge bases of two agents, not only information in conflict, but also information in agreement should be considered. The quantities of conflict and agreement can affect each other. Considering two agents with low quantity of conflict between their knowledge bases, our perception of the degree of conflict between their knowledge bases will be further weakened if they have a lot in common. Furthermore, when two agents have no conflict, it is useful to consider the amount of agreement between them.

We use two quantities of agreement; one is called the quantity of strong agreement which measures the information that both agents “totally” agree with, and the other is called the quantity of weak agreement which measures the information that is believed by one source but is *unknown* to the other. Both quantities will influence the degree of conflict, but their influences are different. Intuitively, the quantity of strong agreement will have more influence on the degree of conflict than the quantity of weak agreement. To illustrate, let us consider the following three knowledge bases: $B_1 = \{p, q\}$, $B_2 = \{\neg p, q\}$, and $B_3 = \{\neg p\}$. B_1 is in conflict with both B_2 and B_3. B_1 and B_2 strongly agree on q, whilst B_1 only weakly agrees with B_3 on q. Clearly the degree of conflict between B_1 and B_2 should be smaller than that between B_1 and B_3 because there is a topic that both B_1 and B_2 agree opon. However, when defining the degree of conflict [18], Hunter did not distinguish the influences of strong agreement and weak agreement. To accompany the definition of degree of conflict, we define a degree of strong agreement and a degree of weak agreement.

It is well-known that priorities or orderings (either implicit or explicit) play an important role in inconsistency handling [1]. The handling of priorities has been shown to be completely in agreement with possibilistic logic [14]. Possibilistic logic [13] is a weighted logic which attaches to each propositional (or first-order) logic formula with a weight belonging to a totally ordered scale, such as $[0, 1]$. An ordering between two formulas is then obtained by comparing the weights attached to them. Possibilistic logic is also known to be a good logical framework for reasoning under inconsistency and uncertainty when only partial information is available. When the weights attached to formulas are all equal to 1, possibilistic logic is reduced to propositional logic. Some measures of information and contradiction in possibilistic logic were also proposed (see [12]). These measures are applied to a single knowledge base in possibilistic logic. In this paper, we consider measures of conflict and agreement between knowledge bases in possibilistic logic. Our measures are defined by generalized prime implicants in possibilistic logic which can be viewed as *partial models* of a possibilistic knowledge base.

The main contributions of the paper are summarized as follows.

- First, we generalize the notion of a prime implicant to a *weighted prime implicant* in possibilistic logic. We show that weighted prime implicants can be used to compile a possibilistic knowledge. A method is given to compute weighted prime implicants of a possibilistic knowledge base.

- Second, we define the notions of *quantity of conflict*, *quantity of strong agreement* and *quantity of weak agreement* using the notion of a weighted prime implicant. We show that these measures satisfy some intuitive properties. When classical knowledge bases are considered, our quantity of conflict coincides with the Dalal distance in propositional logic.

Relationships between two knowledge bases are explored using these measures.

- Third, we define the degree of conflict and two degrees of agreement. We compare our definition of degree of conflict with that given by Hunter in [18] and show that ours is more reasonable in that it takes degree of agreement into account when assigning degree of conflict.

- Finally, we consider two applications of our measures of conflict and agreement to the choice of different combination operators and to the ordering of different knowledge bases.

The rest of the paper is organized as follows. We first give preliminaries on possibilistic logic in Section 2. We then define the notion of a weighted prime implicant in Section 3. Our measures of conflict and agreement between two possibilistic knowledge bases are defined in Section 4. Following this, we discuss the impact of our measures in Section 5. Section 6 discusses related work. In Section 7, we conclude the paper and present some ideas for future work.

2. Preliminaries

2.1. Classical logic

In this paper, we consider a propositional language $\mathcal{L}_{PS}$ from a finite set PS of propositional symbols. The classical consequence relation is denoted as $\vdash$. An interpretation is a total function from PS to $\{true, false\}$. Ω is the set of all possible interpretations. An interpretation w is a model of a formula ϕ iff $w(\phi) = true$. p, q, r,... represent atoms in PS. A literal is an atom p or its negation $\neg p$. We denote literals by l, l_1, .. and formulae in $\mathcal{L}_{PS}$ by ϕ, ψ, γ,... For each formula ϕ, we use $M(\phi)$ to denote its set of models. A *classical knowledge base* B is a finite set of propositional formulae. B is consistent iff there exists an interpretation w such that $w(\phi) = true$ for all $\phi \in B$. A *clause* C is a *disjunction* of literals: $C = l_1 \vee ... \vee l_n$ and its dual clause, or *term* D, is a *conjunction* of literals: $D = l_1 \wedge ... \wedge l_n$. We sometimes also consider a term D as a set of literals. A formula ϕ can be represented by its *conjunctive normal form (CNF)*, which is a conjunction of clauses (or equivalently, a set of clauses), i.e., $CNF_\phi = C_1 \wedge ... \wedge C_m$.

2.2. Possibilistic logic

Possibilistic logic [13] is a weighted logic where each classical logic formula is associated with a level of priority.

The semantics of possibilistic logic is based on the notion of a *possibility distribution* which is a mapping π from Ω to the unit interval [0,1]. The unit interval can be replaced by any totally ordered scale. $\pi(\omega)$ represents the degree of compatibility of the interpretation ω with the available beliefs about the real world. $\pi(\omega) = 0$ means that the interpretation ω is impossible to be the real world, and $\pi(\omega) = 1$ means that nothing prevents ω from being the real world, while $0 < \pi(\omega) < 1$ means that ω is only somewhat possible to be the real world. When $\pi(\omega) > \pi(\omega')$, ω is preferred to ω' for being the real world.

From a possibility distribution π, two measures defined on a set of propositional formulae can be determined. One is the possibility degree of formula ϕ, and is defined as $\Pi_\pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$. The other is the necessity degree of formula ϕ, and is defined as $N_\pi(\phi) = 1 - \Pi_\pi(\neg\phi)$. The possibility degree of ϕ evaluates to what extent ϕ is consistent with knowledge expressed by π and the necessity degree of ϕ evaluates to what extent ϕ is entailed by the available knowledge. $N_\pi(\phi) = 1$ means that ϕ is a totally certain piece of knowledge, while $N_\pi(\phi) = 0$ expresses the complete lack of knowledge of priority about ϕ, but does not mean that ϕ is or should be false. We have $N_\pi(true) = 1$ and $N_\pi(\phi \wedge \psi) = min(N_\pi(\phi), N_\pi(\psi))$ for all ϕ and ψ.

At the syntactic level, a formula, called a *possibilistic formula*, is represented by a pair (ϕ, a) where ϕ is a formula and $a \in [0, 1]$. The possibilistic formula (ϕ_i, a_i) means that the necessity degree of ϕ_i is at least equal to a_i, i.e. $N(\phi_i) \geq a_i$. Pieces of uncertain information from a source are represented by a *possibilistic knowledge base* which is a finite collection of possibilistic formulas of the form $B = \{(\phi_i, a_i) : i = 1, ..., n\}$. In this paper, we only consider possibilistic knowledge bases where every formula ϕ is a classical propositional formula. The classical base associated with B is denoted as B^*, namely $B^* = \{\phi_i | (\phi_i, a_i) \in B\}$. A possibilistic knowledge base B is consistent if and only if its classical base B^* is consistent. In this paper, we consider a classical knowledge base as a particular possibilistic knowledge base where every formula has weight 1.

Given a possibilistic knowledge base B, a unique *possibility distribution*, denoted as π_B, can be obtained by the principle of minimum specificity. For all $\omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall(\phi_i, a_i) \in B, \omega \models \phi_i, \\ 1 - max\{a_i|\omega \not\models \phi_i\} & \text{otherwise.} \end{cases} \quad (1)$$

The *inconsistency degree* of B, which defines the level of inconsistency of B, is defined by

$$Inc(B) = 1 - max_\omega \pi_{\mathcal{B}}(\omega).$$

Definition 1. *[13] Let B be a possibilistic knowledge base, and $a \in [0,1]$. We call the a-cut (respectively strict a-cut) of B, denoted as $B_{\geq a}$ (respectively $B_{>a}$), the set of classical formulas in B having a necessity degree at least equal to a (respectively strictly greater than a).*

Let B and B' be two possibilistic knowledge base, B and B' are said to be equivalent, denoted $B \equiv_s B'$, if and only if $\forall\ a \in (0,1]$, $B_{\geq a} \equiv B'_{\geq a}$.

The inconsistency degree of B in terms of the a-cut can be equivalently defined as [13]:

$$Inc(B) = max\{a_i|B_{\geq a_i} \textit{ is inconsistent}\}.$$

Definition 2. *[13] Let B be a possibilistic knowledge base. Let $(\phi,\ a)$ be a possibilistic formula with $a > Inc(B)$. (ϕ, a) is said to be a consequence of B, denoted by $B \vdash_\pi (\phi, a)$, iff $B_{\geq a} \vdash \phi$. Given two possibilistic knowledge bases B_1 and B_2, $B_1 \vdash_\pi B_2$ iff $B_1 \vdash (\phi, a)$ for all $(\phi, a) \in B_2$.*

3. Weighted Prime Implicant

In this section, we first define the notion of a weighted prime implicant of a possibilistic knowledge base and discuss its properties. We then propose an algorithm for computing the weighted prime implicants of a possibilistic knowledge base. In the following, we assume that knowledge bases are individually consistent.

3.1. Definition and properties

A term D is an implicant of formula ϕ iff $D \vdash \phi$ and D does not contain two complementary literals. D is an implicant of a knowledge base B if it is an implicant of $\wedge_{\phi_i \in B} \phi_i$.

Definition 3. *[9] A* prime implicant *of knowledge base B is an implicant D of B such that for every other implicant D' of B, $D \not\vdash D'$.*

Prime implicants are often used in knowledge compilation [9] to make the deduction tractable. Suppose D_1, ..., D_k are all the prime implicants of B, we have $B \vdash \phi$, for any ϕ iff for every prime implicant D_i, $D_i \vdash \phi$.

Now we define weighted prime implicants of a possibilistic knowledge base. Let us first define weighted prime implicants for possibilistic knowledge base $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ where ϕ_i are clauses. For a more general possibilistic knowledge base, we can decompose it to an equivalent possibilistic knowledge base whose formulas are clauses by the min-decomposability of necessity measures, i.e., $N(\wedge_{i=1,k}\phi_i) \geq m \Leftrightarrow \forall i, N(\phi_i) \geq m$. For example, a possibilistic formula of the form $(\phi_1 \wedge ... \wedge \phi_n, a)$ can be equivalently decomposed into a set of formulas (ϕ_1, a),...,(ϕ_n, a).

Let $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ be a possibilistic knowledge base where ϕ_i are clauses. A weighted implicant of B is $D = \{(\psi_1, b_1), ..., (\psi_k, b_k)\}$[2], a possibilistic knowledge base, such that $D \vdash_\pi B$, where ψ_i are literals such that no two complementary literals exist in D and $\psi_i \neq \psi_j$ for $i \neq j$. Let D and D' be two weighted implicants of B, D is said to be *subsumed* by D' if and only if $D \neq D'$, $D'^* \subseteq D^*$ and for any $(\psi_i, a_i) \in D'$, $\exists (\psi_i, b_i) \in D$ with $a_i \leq b_i$. In other words, D is subsumed by D' if and only if $D \neq D'$, and every literal appearing in D' must appear in D with higher or same necessity degree. Alternatively, this subsumption relation can be defined by possibilistic inference. This is shown as follows.

Proposition 1. *Let D and D' be two weighted implicants of B. D is subsumed by D' if and only if $D \neq D'$ and $D \vdash_\pi D'$.*

Example 1. *Let $B = \{(q \vee r, 0.9), (p \wedge q, 0.8), (q, 0.8), (q \vee \neg s, 0.6)\}$ be a possibilistic knowledge base. It is easy to check that $D = \{(p, 0.8), (q, 0.9), (r, 0.9)\}$ and $D' = \{(p, 0.8), (q, 0.8), (r, 0.9)\}$ are two weighted implicants of B, and D is subsumed by D'.*

According to Example 1, a weighted implicant can be logically too strong, that is, it may be subsumed by another weighted implicant. In the following,

[2] In this paper, to simplify notations, we use D to denote both a term and a set of literals attached with necessity degree.

we define a weighted prime implicant which generalizes the notion of a prime implicant.

Definition 4. *Let* $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ *be a possibilistic knowledge base where* ϕ_i *are clauses. A weighted prime implicant of* B *is a possibilistic knowledge base* D *such that*

1. D *is a weighted implicant of* B
2. *There does not exist other weighted implicant* D' *of* B *such that* D *is subsumed by* D'*.*

A weighted prime implicant of a possibilistic knowledge base is one of its weighted implicants that is not subsumed by another weighted implicant of it.

Example 2. *(Continue Example 1) The weighted prime implicants of* B *are* $D_1 = \{(p, 0.8), (q, 0.9)\}$*,* $D_2 = \{(q, 0.8), (r, 0.9)\}$ *and* $D_3 = \{(r, 0.8), (q, 0.9)\}$*.*

We are able to show the following corollary by Proposition 1.

Corollary 1. *Let* B *be a possibilistic knowledge base. Then* D *is a weighted prime implicant of* B *if and only if it satisfies the following two conditions.*

1. D *is a weighted implicant of* B
2. *For every other weighted implicant* D' *of* B*,* $D \not\vdash_\pi D'$*.*

The proof of Corollary 1 is easy to establish. It shows that a weighted prime implicant can be equivalently defined as a logically weakest weighted implicant.

We show that the weighted prime implicant generalizes the prime implicant.

Proposition 2. *Let* $B = \{(\phi_1, 1), ..., (\phi_n, 1)\}$ *be a possibilistic knowledge base which contains formulas with weight 1, i.e.,* B *is a classical knowledge base. Then* D *is a weighted prime implicant of* B *iff* D *is a prime implicant of* B*.*

However, given possibilistic knowledge base B, if D is a weighted prime implicant of B, then D^* is not necessarily a prime implicant of B^*. A counterexample can be found in Example 2, where D_2 is a weighted prime implicant, but $D_2^* = \{p, q, r\}$ is not a prime implicant of B^*.

The following proposition says that weighted prime implicants can be used to compile a possibilistic knowledge base.

Proposition 3. *Let B be a possibilistic knowledge base. If D_1,...,D_n are all the weighted prime implicants of B, then for any formula ϕ and any weight $a \in [0,1]$, we have,*

$$B \vdash_{\pi} (\phi, a) \; iff \; D_i \vdash_{\pi} (\phi, a), \; for \; all \; D_i.$$

Our method of compiling possibilistic knowledge bases is different from other methods given in [5] and [6]. Our method compiles a possibilistic knowledge base into a set of weighted prime implicants which are also possibilistic knowledge bases. In contrast, their methods transform a possibilistic knowledge base to a propositional knowledge base by introducing some new propositional symbols.

Next we give some justification for the weighted prime implicants.

First of all, to measure information in a single classical knowledge base (this knowledge base may be inconsistent), most of the current methods are based on the *models* of the knowledge base [16, 25]. In [16], the degree of inconsistency is measured based on the *model* of an inconsistent knowledge base in the framework of quasi-classical logic. In [25], a *quasi-model* of an inconsistent knowledge base, which is a maximal consistent subbase of the knowledge base, is defined to measure information for inconsistent sets. By Definition 4, each weighted prime implicant can be interpreted as a partial truth assignment. Suppose p is an atom and D is a weighted prime implicant, then $(p, a) \in D$ means that there is an argument for p in D with certainty degree a, and $(\neg p, b) \in D$ means that there is an argument against p in D with certainty degree b, while $\phi \notin D^*$ means the truth value of ϕ is undetermined in D. By Proposition 3, a weighted prime implicant can be viewed as a *partial model* of a possibilistic knowledge base. This is consistent with the methods in [16, 25].

Second, when all the formulas in a possibilistic knowledge base have the same weight 1, a weighted prime implicant is a prime implicant. In classical logic, a classical model is often used to define the relationships between two knowledge bases, such as the distance between two knowledge bases [11]. However, classical models are not suitable for us to define the quantities of agreement between knowledge bases because a classical model must assign a truth value to every atom in the knowledge bases. Let us look at the example in the introduction again. The only model for B_1 is $w = \{p, q\}$ and there are two models for B_3, i.e., $w_1 = \{\neg p, q\}$ and $w_2 = \{\neg p, \neg q\}$. B_1 and B_3 weakly agree on q because only B_1 supports q. However, by comparing w with w_1 or comparing w with w_2 we cannot get such a conclusion. In contrast, a prime implicant can be viewed as a *partial* truth assignment. That is, only some of the atoms are assigned truth values. Given a prime implicant D of B, a three-value semantics can be associated with it as follows:

$$v_D(p) = \begin{cases} true & \text{if } D \vdash p, \\ false & \text{if } D \vdash \neg p, \\ undetermined & \text{otherwise.} \end{cases} \quad (2)$$

That is, if an atom does not appear in a prime implicant, then its truth value w.r.t. the three-valued interpretation associated with the prime implicant is undetermined. Thus, a prime implicant does not specify the truth values of some atoms. Given two prime implicants D_1 and D_2, if an atom appears in a prime implicant and does not appear in another one, then it has truth value *true* w.r.t. v_{D_1} and has true value *unknown* w.r.t. v_{D_2}. Since the truth value of the atom w.r.t. v_{D_2} can be either true or false, we can say that D_1 and D_2 weakly agrees on p. In previous example, B_1 has one prime implicant $D_1 = \{p, q\}$ and B_3 has one prime implicant $D_2 = \{\neg p\}$, where D_2 does not contain any information on q; so the quantity of weak agreement between D_1 and D_2 is 1. As a consequence, the weak agreement between B_1 and B_3 is 1, which is consistent with our analysis above.

3.2. A method for computing weighted prime implicants

We give another important property of weighted prime implicants. We first define a new notion of prime implicants. Given a propositional knowledge base B and a term D that does not contain two complementary literals, a term D_i is called a D-extended implicant of B if and only if $D \cup D_i \vdash \phi$ for any $\phi \in B$, and D_i does not contain two complementary literals. A D-extended prime implicant D_i of B is a D-extended implicant of B such that for every other D-extended implicant D_j of B, $D \cup D_i \not\vdash D \cup D_j$. When $D = \emptyset$, then a D-extended prime implicant of a propositional knowledge B is simply a prime implicant of B. So, any algorithm for computing D-extended implicants of a propositional knowledge base can be used to compute prime implicants of a propositional knowledge base. Given a possibilistic knowledge base B, we define $B_{=a} = \{\phi_i : (\phi_i, a) \in B\}$.

Proposition 4. *Given a possibilistic knowledge base B, suppose D is a set of weighted literals such that no two complementary literals exist in D. Suppose $b_1, ..., b_k$ are all the distinct weights appearing in D such that $b_i > b_j$ for*

$i < j$. Then D is a weighted prime implicant of B if and only if $D_{=b_i}$ is a $D_{>b_i}$-extended prime implicant of $B_{=b_i}$ for $1 \leq i \leq k$.

According to Proposition 4, we are able to provide an algorithm for computing weighted prime implicants.

Algorithm 1

Input: a possibilistic knowledge base $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$.

Output: a set of weighted prime implicants of B.

begin

1. Let $b_1, ..., b_k$ be all the distinct weights appearing in D such that $b_i > b_j$ for $i < j$;
2. $h := 1$; $\mathsf{W} := \{\emptyset\}$; $\mathsf{W}_{temp} := \emptyset$;
3. **while** $h \leq k$ **do**
4. **for each** $D \in \mathsf{W}$
   $\mathsf{W}_D := \{D_i : D_i \text{ is a } D^*\text{-extended prime implicant of } B_{=b_h}\}$
   $\mathsf{W}_{temp} := \mathsf{W}_{temp} \cup \{D \cup \{(\psi_i, b_h) : \psi_i \in D_i\} : D_i \in \mathsf{W}_D\}$
5. $h = h + 1$;
6. $\mathsf{W} := \mathsf{W}_{temp}$;
7. $\mathsf{W}_{temp} := \emptyset$;
8. **end-while**
9. **return** W.

end

Assume that $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ and $b_1, ..., b_k$ are all the distinct weights appearing in B such that $b_i > b_j$ for $i < j$. We stratify B as $B = B_{=b_1} \cup ... \cup B_{=b_k}$, where $B_{=b_i} = \{\phi_j : (\phi_j, b_i) \in B \text{ for } i = 1, ..., k\}$. In Algorithm 1, we first find all prime implicants of $B_{=b_1}$ and all literals in each of these prime implicants are attached with weight b_1. For each of these prime implicants D, we find all the D-extended prime implicants of $B_{=b_2}$ and all the literals in any of these D-extended prime implicants are attached with weight b_2. A D-extended prime implicant can be considered as an incomplete weighted prime implicant. We continue this process to extend each incomplete weighted prime implicant until $B_{=b_k}$. According to Proposition 4, it is not difficult to verify that Algorithm 1 will return all weighted prime implicants of B. In the i-th iteration of the while loop, we need to compute all D-extended prime implicants of $B_{=b_i}$ for any $D \in \mathsf{W}$. Since computing D-extended prime implicants of a propositional knowledge base is at least as hard as computing prime implicants of a propositional knowledge base, computing weighted prime implicants of a possibilistic knowledge base using our algorithm is at least as hard as computing prime implicants of a propositional knowledge base.

Now the problem is that, given a term D, how to compute all D-extended prime implicants of a propositional knowledge base K. This problem can be solved by adapting existing methods for computing prime implicants of a propositional knowledge base. We adapt the syntactic method given in [8], where a dual transformation algorithm was proposed to compute the set of prime implicants of a propositional formula ϕ represented by CNF_ϕ. A *conjunctive quantum* is defined as a pair (l, F), where l is a literal and F is its set of *conjunctive coordinates* w.r.t. CNF_ϕ that contains the subset of clauses in CNF_ϕ to which l belongs. Note that if l does not occur in CNF_ϕ, then $F = \emptyset$. For simplicity, each quantum is denoted as l^F.

We now adapt the method given in [8]. Let $\phi = \wedge_{\phi_i \in K}\phi_i$ and $D = l_1 \wedge ... \wedge l_n$. Suppose $D' = l_{n+1} \wedge ... \wedge l_{n+m}$ is a term of CNF_ϕ. Remember that a term can be equivalently taken as a set of literals. $D \cup D'$ can be represented as $D \cup D' = \{l_1^{F^1}, ..., l_{n+m}^{F^{n+m}}\}$, where F^i is the set of conjunctive coordinates of l_i w.r.t. CNF_ϕ. It is not difficult to see that D' is a D-extended implicant of ϕ if $\cup_{i=1}^{n+m} F^i = CNF_\phi$ and there is no pair of contradictory literals in $\{l_i : i = 1, ..., n+m\}$. Let us define $\hat{F}^i = F^i - \cup_{j=1, j \neq i}^{n+m} F^i$. Then D' is a D-extended prime implicant of ϕ if it satisfies the *non redundancy* condition, i.e. $\forall i \in \{n+1, ..., n+m\}$, $\hat{F}^i \neq \emptyset$.

The modified dual transformation algorithm first calculates the conjunctive coordinates of all the literals of CNF_ϕ and D, and then begins a search in a state space where each state is represented by a set of quanta that represents an incomplete D-extended prime implicant. The initial set of states is $\{D_1, ..., D_k\}$, where $D_i = \{l_1^{F^1}, ..., l_n^{F^n}, (l')_i^{F^{l'}}\}$, where l'_i belongs to one specific clause $C \in CNF_\phi$ with $C = (l')_1 \vee ... \vee (l')_k$. Successor of each state in the state space is generated by adding a new conjunctive quantum to the state which is consistent with the quanta in the state and the new state still satisfies the non redundancy condition. For each state $D_q = \{l_1^{F^1}, ..., l_m^{F^m}\}$ for $m > n$, we define $G_{D_q} = CNF_\phi \setminus \cup_{i=1}^m F^i$. For each state space, if we get a state D_q such that $G_{D_q} = \emptyset$, then $D_q \setminus D$ is a D-extended prime implicant of ϕ and we call D_q a final state. It is clear that all of the D-extended prime implicants of ϕ can be found after we get all the final states.

Let us look at an example to illustrate Algorithm 1.

Example 3. *(Continue Example 1) There are three different weights appearing in B, so $k = 3$, and $b_1 = 0.9$, $b_2 = 0.8$ and $b_3 = 0.6$. Let $h = 0$*

and $\mathsf{W}=\{\emptyset\}$. *We have* $h \leq k$. *It is easy to check that* $\mathsf{W}_{\emptyset} = \{\{q\},\{r\}\}$ *and* $\mathsf{W}_{\mathsf{temp}} = \{\{(q,0.9)\},\{(r,0.9)\}\}$. *Next,* $h = 1$ *and we go to the second iteration of the while loop. There are two sets in* W. *We show how to get* $\mathsf{W}_{\{(\mathsf{q},0.9)\}}$, *i.e. the set of* $\{q\}$*-extended prime implicants of* $B_{=0.8}$. *It is easy to see that* $B_{=0.8} = \{p \vee r, q\}$. *Let* $D = \{q\}$ *and* $\phi = (p \vee r) \wedge q$, *which is in CNF form. Let* $C_1 = p \vee r$ *and* $C_2 = q$. *The literals that occur in* ϕ *and* D *can be represented by the following conjunctive quanta:*

$$p^{\{C_1\}},\ q^{\{C_2\}},\ r^{\{C_1\}}.$$

The initial set of states is $\{D_1, D_2\}$, *where* $D_1 = \{q^{\{C_2\}}, p^{\{C_1\}}\}$ *and* $D_2 = \{q^{\{C_2\}}, r^{\{C_1\}}\}$. *Since* $G_{D_i} = \emptyset$ *for* $i = 1,2$, *we conclude that both* D_1 *and* D_2 *are final states and there are no other final states. So we return* $D_1 \setminus D$ *and* $D_2 \setminus D$ *as two* D*-extended prime implicants of* $B_{=0.8}$. *Therefore,* $\mathsf{W}_{\{(\mathsf{q},0.9)\}} = \{\{p\},\{r\}\}$ *and* $\mathsf{W}_{\mathsf{temp}} = \{\{(q,0.9),(p,0.8)\},\{(q,0.9),(r,0.8)\}\}$. *Similarly, we can get* $\mathsf{W}_{\{(\mathsf{r},0.9)\}} = \{\{p\},\{q\}\}$ *and* $\mathsf{W}_{\mathsf{temp}} = \{\{(q,0.9),(p,0.8)\},\{(q,0.9),(r,0.8)\},\{(r,0.9),(q,0.8)\}\}$ *and* $\mathsf{W} = \mathsf{W}_{\mathsf{temp}}$. $h = 2$ *and we go to the third iteration of the while loop. It is easy to check that for any* $D \in \mathsf{W}$, $\mathsf{W}_{\mathsf{D}} = \emptyset$. *So* W *is not changed when* $h = 3$. *So we get all the weighted prime implicants of* B*:* $\{(q,0.9),(p,0.8)\}$, $\{(q,0.9),(r,0.8)\}$, $\{(r,0.9),(q,0.8)\}$. *This result coincides with the result obtained in Example 2.*

4. Measures of Conflict and Agreement Between Two Possibilistic Knowledge Bases

4.1. Quantity of conflict and quantities of agreement

In this subsection, we measure the quantities of conflict and agreement between two possibilistic knowledge bases based on the weighted prime implicant. We then define the corresponding degree of conflict and degrees of agreement in the next subsection.

Before defining the quantity of conflict between two knowledge bases, we need to define the quantity of conflict between two weighted prime implicants. This is inspired by the definition of Dalal distance between two knowledge base [11] which is defined by the Hamming distance between two interpretations.

We use a notation given in [16]. Let p be a propositional symbol, and $\sim$ be the complementation operation defined as $\sim p$ is $\neg p$ and $\sim(\neg p)$ is p. This operation is not in the object language but will be used to make definitions clearer.

Given two possibilistic knowledge bases B_1 and B_2, suppose C and D are weighted prime implicants for B_1 and B_2 respectively, then the quantity of conflict between C and D is computed as follows. For each pair of complementary literals which belong to C and D respectively, we take the minimum of their necessity degrees. We then sum up all the values taken from the minimum of necessity degrees of the complementary literals. More formally, we have the following definition.

Definition 5. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose C is a weighted prime implicant of B_1 and D is a weighted prime implicant of B_2, then the quantity of conflict between C and D is defined as*

$$q_{Con}(C, D) = \sum_{(l,a)\in C \text{ and } (\sim l,b)\in D} min(a, b). \tag{3}$$

When the weights associated with all the formulas are 1, $q_{Con}(C, D)$ is the cardinality of the set of atoms which are in conflict in $C \cup D$. In this case, the quantity of conflict is similar to the Hamming distance between two interpretations.

The quantity of conflict between two possibilistic knowledge base is the minimal quantity of conflict between their respective weighted prime implicants.

Definition 6. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose $\mathcal{C}$ is the set of weighted prime implicants of B_1 and $\mathcal{D}$ is the set of weighted prime implicants of B_2, then the quantity of conflict between B_1 and B_2 is defined as*

$$Q_{Con}(B_1, B_2) = min\{q_{Con}(C, D) | C \in \mathcal{C},\ D \in \mathcal{D}\}. \tag{4}$$

The quantity of conflict between B_1 and B_2 measures how much information is in conflict between B_1 and B_2. In Equation 4, we use the minimum operator to aggregate the quantities of conflict between weighted prime implicants in $\mathcal{C}$ and weighted prime implicants in $\mathcal{D}$. That is, a knowledge base B_1 is in conflict with another one B_2 to the degree $Q_{Con}(B_1, B_2)$ if for each pair of weighted prime implicants in B_1 and B_2 respectively, their quantity of conflict is at least $Q_{Con}(B_1, B_2)$.

Example 4. *Let* $B_1 = \{(\neg p, 0.8), (\neg q \vee r, 0.6)\}$ *and* $B_2 = \{(p \vee \neg r, 0.7), (q, 0.5)\}$ *be two possibilistic knowledge bases. The set of weighted prime implicants of* B_1 *is* $C_1 = \{(\neg p, 0.8), (\neg q, 0.6)\}$ *and* $C_2 = \{(\neg p, 0.8), (r, 0.6)\}$*, and the set of weighted prime implicants of* B_2 *is* $D_1 = \{(p, 0.7), (q, 0.5)\}$ *and* $D_2 = \{(\neg r, 0.7), (q, 0.5)\}$*. It is easy to calculate that* $q_{Con}(C_1, D_1) = 1.2$*,* $q_{Con}(C_1, D_2) = 0.5$*,* $q_{Con}(C_2, D_1) = 0.7$*,* $q_{Con}(C_2, D_2) = 0.6$*. Therefore, the quantity of conflict between* B_1 *and* B_2 *is 0.5.*

It happens that the quantity of conflict between B_1 and B_2 in Example 4 is the same as the inconsistency degree of $B_1 \cup B_2$, which is the maximal weight a in $B_1 \cup B_2$ such that the a-cut of $B_1 \cup B_2$ is inconsistent. However, this result does not hold in general. Consider $B_1 = \{(\neg p, 0.8), (r, 0.7)\}$ and $B_2 = \{(p, 0.7), (\neg r, 0.6)\}$, we have $Q_{Con}(B_1, B_2) = 1.3$ but $Inc(B_1 \cup B_2) = 0.6$.

We consider some properties of the measure Q_{Con}.

Proposition 5. *Let* B_1 *and* B_2 *be two possibilistic knowledge bases. Then* $B_1 \cup B_2$ *is consistent iff* $Q_{Con}(B_1, B_2) = 0$*.*

According to Definition 6 and Proposition 5, two knowledge bases B_1 and B_2 are inconsistent with each other if and only if each weighted prime implicant of B_1 is inconsistent with each weighted prime implicant of B_2.

Proposition 6. *Let* B*,* B_1 *and* B_2 *be three possibilistic knowledge bases. If* $B_2 \vdash_\pi B_1$*, then* $Q_{Con}(B, B_1) \leq Q_{Con}(B, B_2)$*.*

Proposition 6 tells us that the quantity of conflict between two knowledge bases increases when one of them has been replaced by a logically stronger possibilistic knowledge base.

We have shown in [30] that function Q_{Con} is syntax-independent.

Proposition 7. *Given four possibilistic knowledge bases* B_1*,* B_1'*,* B_2 *and* B_2'*, suppose* $B_1 \equiv_s B_1'$ *and* $B_2 \equiv_s B_2'$*, then* $Q_{Con}(B_1, B_2) = Q_{Con}(B_1', B_2')$*.*

When we consider knowledge bases in classical propositional logic, the quantity of conflict is consistent with the Dalal distance between two knowledge bases [11]. The Dalal distance between two models w_i, w_j of a classical formula is the Hamming distance between them, i.e., $Dalal(w_i, w_j) = |w_i - w_j| + |w_j - w_i|$. The Dalal distance between two knowledge bases is the minimal value of distances between all the pairs of models, one for each knowledge base.

Let X be a set of classical propositional formulas. Let $I(X)$ be the set of interpretations of X delineated by the atoms used in X (i.e. $I(X) = 2^{Atoms(X)}$, where $Atom(X)$ denotes the set of atoms appearing in X). Let $M(X,Y)$ be the set of **models** of X that are in $I(Y)$. That is, $M(X,Y) = \{w \models \wedge X | w \in I(Y)\}$.

Proposition 8. *Let B_1 and B_2 be two classical propositional knowledge bases that are consistent. Let Dalal $(B_1, B_2) = min\{Dalal(w_i, w_j) | w_i \in M(B_1, B_1 \cup B_2), w_j \in M(B_2, B_1 \cup B_2)\}$. Then we have*

$$Q_{Con}(B_1, B_2) = Dalal(B_1, B_2).$$

Proposition 8 is very important, because it tells us that our quantity of conflict coincides with the Dalal distance in classical propositional logic. Therefore, the quantity of conflict $Q_{Con}(B_1, B_2)$ in prioritized case can be taken as a generalization of the Dalal distance between two knowledge bases.

There are two different kinds of quantities of agreement: the quantity of strong agreement and the quantity of weak agreement. Intuitively, the quantity of strong agreement measures how much information is supported by both knowledge bases and the quantity of weak agreement measures the information supported by only one knowledge base and is *unknown* to the other. We first define the quantity of strong agreement.

Definition 7. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose C is a weighted prime implicant of B_1 and D is a weighted prime implicant of B_2, then the quantity of strong agreement between C and D is defined as*

$$q_{SA}(C, D) = \sum_{(l,a) \in C,\ (l,b) \in D} min(a, b). \tag{5}$$

When the weights associated with all the formulas are 1, $q_{SA}(C, D)$ is the cardinality of the set of literals which are in both C and D.

Definition 8. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose $\mathcal{C}$ is the set of weighted prime implicants of B_1 and $\mathcal{D}$ is the set of weighted prime implicants of B_2, then the quantity of strong agreement between B_1 and B_2 is defined as*

$$Q_{SA}(B_1, B_2) = max\{q_{SA}(C, D) | C \in \mathcal{C},\ D \in \mathcal{D}\}. \tag{6}$$

The quantity of strong agreement between B_1 and B_2 is the maximal quantity of strong agreement between weighted prime implicants of B_1 and B_2 respectively. Dual to the quantity of conflict, we use the maximum to aggregate the quantities of strong agreement between weighted prime implicants in $\mathcal{C}$ and weighted prime implicants in $\mathcal{D}$.

Example 5. *Let $B_1 = \{(p \vee q, 0.7), (s, 0.6)\}$ and $B_2 = \{(p, 0.8), (s \vee r, 0.6)\}$ be two possibilistic knowledge bases. B_1 has two weighted prime implicants $C_1 = \{(p, 0.7), (s, 0.6)\}$ and $C_2 = \{(q, 0.7), (s, 0.6)\}$, and B_2 has two weighted prime implicants $D_1 = \{(p, 0.8), (s, 0.6)\}$ and $D_2 = \{(p, 0.8), (r, 0.6)\}$. By Equation 5, we have $q_{SA}(C_1, D_1) = 1.3$, $q_{SA}(C_1, D_2) = 0.7$, $q_{SA}(C_2, D_1) = 0.6$, and $q_{SA}(C_2, D_2) = 0$. Therefore, the quantity of strong agreement between B_1 and B_2 is $Q_{SA}(B_1, B_2) = 1.3$.*

There exist B, B_1 and B_2 such that $B_2 \vdash_\pi B_1$ but $Q_{SA}(B, B_1) > Q_{SA}(B, B_2)$. This is because if an agent gets more information, some information that it strongly agrees with another agent may be lost. For example, suppose $B = \{(p, 0.9)\}$, $B_1 = \{(p \vee q, 0.8)\}$ and $B_2 = \{(p \vee q, 0.8), (\neg p, 0.7)\}$. Then we have $Q_{SA}(B, B_1) = 0.8$ and $Q_{SA}(B, B_2) = 0$. Therefore, if one agent gets more information, the quantity of strong agreement between its knowledge base and that of another agent may decrease.

The function Q_{SA} is also syntax-independent.

Proposition 9. *Given four possibilistic knowledge bases B_1, B_1', B_2 and B_2', suppose $B_1 \equiv_s B_1'$ and $B_2 \equiv_s B_2'$, then $Q_{SA}(B_1, B_2) = Q_{SA}(B_1', B_2')$.*

Proof of Proposition 9 is similar to that of Proposition 7.

The quantity of strong agreement measures the information that both agents "totally" agree with. Alternatively, we may want to measure the information that is believed by one source but is *unknown* to the other. This is achieved by defining the quantity of weak agreement.

Definition 9. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose C is a weighted prime implicant of B_1 and D is a weighted prime implicant of B_2, then the quantity of weak agreement between C and D is defined as*

$$q_{WA}(C, D) = \sum_{(l_i, a_i) \in C \cup D, l_i \notin C^* \cap D^* \text{ and } \sim l_i \notin C^* \cup D^*} a_i. \tag{7}$$

That is, the quantity of weak agreement between weighted prime implicants C and D of B is the sum of weights of literals appearing in either C or D which do not appear in both C and D and whose complements appear in neither C nor D. When the weights associated with all the formulas are 1, $q_{WA}(C, D)$ is the cardinality of the set of literals which are in only one of C and D but not both.

Definition 10. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose $\mathcal{C}$ is the set of weighted prime implicants of B_1 and $\mathcal{D}$ is the set of weighted prime implicants of B_2, then the quantity of weak agreement between B_1 and B_2 is defined as*

$$Q_{WA}(B_1, B_2) = max\{q_{WA}(C, D)|C\in\mathcal{C},\ D \in \mathcal{D}\}. \tag{8}$$

The quantity of weak agreement between B_1 and B_2 is the maximal quantity of weak agreement between weighted prime implicants of B_1 and B_2 respectively.

Example 6. *(Continue Example 4) By Equation 7, we have $q_{WA}(C_1, D_1) = 0$, $q_{WA}(C_1, D_2) = 1.5$, and $q_{WA}(C_2, D_1) = 1.1$, $q_{WA}(C_2, D_2) = 1.3$. We illustrate how to compute $q_{WA}(C_1, D_2)$. q belong to both C_1 and D_2, so we do not consider it. Both p and $\neg r$ only appear in one of C_1 and D_2 and their complements do not appear $C_1 \cup D_2$. Therefore, $q_{WA}(C_1, D_2)$ is the sum of their weights, i.e., 1.5. According to Definition 10, the quantity of weak agreement between B_1 and B_2 is $Q_{WA}(B_1, B_2) = 1.5$.*

Similar to the function Q_{SA}, there exist B, B_1 and B_2 such that $B_2 \vdash_\pi B_1$ but $Q_{WA}(B, B_1) > Q_{WA}(B, B_2)$. This can be explained intuitively as follows. If an agent gets more information, her knowledge base B_1 is changed to B_2 which is logically stronger than B_1, then some information that she weakly agrees with another agent whose knowledge base is B may become conflicting. For example, let $B_1 = \{(p, 1)\}$ and $B = \{(q, 1)\}$, then $Q_{WA}(B, B_1) = 2$. However, the quantity of weak agreement between $B_2 = \{(p, 1), (p \to \neg q, 1)\}$ and B is $Q_{WA}(B, B_2) = 1$, where $B_1 \subseteq B_2$.

Proposition 10. *Given four possibilistic knowledge bases B_1, B'_1, B_2 and B'_2, suppose $B_1 \equiv_s B'_1$ and $B_2 \equiv_s B'_2$, then $Q_{WA}(B_1, B_2) = Q_{WA}(B'_1, B'_2)$.*

Proof of Proposition 10 is similar to that of Proposition 7.

Based on the quantity of conflict and quantities of agreement, we can define the following relationships between two knowledge bases.

- two knowledge bases B_1 and B_2 are said to be *totally in conflict* if and only if $Q_{Con}(B_1, B_2) > 0$ and $Q_{SA}(B_1, B_2) = Q_{WA}(B_1, B_2) = 0$. For example, $B_1 = \{(p, 0.5), (q, 0.6)\}$ and $B_2 = \{(\neg p, 0.7), (\neg q, 0.4)\}$ are totally in conflict.

- two knowledge bases B_1 and B_2 are *totally in agreement* if and only if $Q_{Con}\ (B_1, B_2) = Q_{WA}(B_1, B_2) = 0$ and $Q_{SA}(B_1, B_2) > 0$. For example, $B_1 = \{(p, 0.7)\}$ and $B_2 = \{(p, 0.8)\}$ are totally in agreement.

- two knowledge bases B_1 and B_2 are *partially in conflict* if and only if $Q_{Con}(B_1,\ B_2) > 0$ and $Q_{SA}(B_1, B_2) + Q_{WA}(B_1, B_2) > 0$. In Example 4, B_1 and B_2 are partially in conflict.

4.2. Degree of conflict and degrees of agreement

In this subsection, we will define a degree of conflict and two degrees of agreement between two possibilistic knowledge bases based on quantities of conflict and agreement defined above.

The degree of conflict measures to what extent two knowledge bases are in conflict. It was first introduced in [18] to measure the believability of arguments.

Definition 11. *Let B_1 and B_2 be two knowledge bases, and let $Dalal(B_1, B_2)$ be the Dalal distance between B_1 and B_2 defined in Proposition 8. The degree of conflict between B_1 and B_2, denoted as $C(B_1, B_2)$, is defined as follows:*

$$C(B_1, B_2) = \frac{Dalal(B_1, B_2)}{log_2(|I(B_1 \cup B_2)|)} \tag{9}$$

Although this definition gives a method to measure the degree of conflict, it can sometimes overestimate the degree of conflict between two knowledge bases. The problem with Hunter's definition is that it does not differentiate the influence of quantity of strong agreement and quantity of weak agreement. For example, let us consider two pairs of knowledge bases (B_1, B_2) and (B_1', B_2), where $B_1 = \{p, q, r\}$, $B_2 = \{\neg p, q, r\}$ and $B_1' = \{p\}$. Although the quantity of conflict between B_1 and B_2 is 1, the quantity of strongly agreement between them is 2. That means B_1 and B_2 have more opinion in common than that in conflict. In contrast, the quantity of conflict between B_1' and B_2 is still 1, but now $Q_{SA}(B_1', B_2) = 0$ and $Q_{WA}(B_1', B_2) = 2$. The degree of conflict between B_1' and B_2 should be higher than the degree of conflict between B_1 and B_2. However, by Equation 9, $C(B_1, B_2) = C(B_1', B_2) = 1/3$. This is not reasonable.

According to the above analysis, the quantity of strong agreement and quantity of weak agreement should have different influence on the degree of conflict. We propose a degree of conflict as follows.

Definition 12. *Let B_1 and B_2 be two possibilistic knowledge bases. Let C be a weighted prime implicant of B_1 and D be a weighted prime implicant of B_2. $Atom_{Con}(C, D)$ denotes the cardinality of the set of atoms which are in conflict in $C \cup D$. Then the degree of conflict between C and D is defined as*

$$d_{Con}(C, D) = \frac{q_{Con}(C, D)}{Atom_{Con}(C, D) + q_{SA}(C, D) + \lambda q_{WA}(C, D)}, \tag{10}$$

where $\lambda \in (0, 1]$ is used to weaken the influence of the quantity of weak agreement on the degree of conflict.

By default, we set $\lambda = 0.5$, that is, the quantity of weak agreement only has "half" of the influence on the degree of conflict as the quantity of strong agreement. The rationale for this setting is that when computing the quantity of strong agreement between two weighted prime implicants, two appearances of a literal in both weighted prime implicants are counted as 1, whilst when computing the quantity of weak agreement between weighted prime implicants, one appearance of a literal in each weighted prime implicant is counted as 1.

Definition 13. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose $\mathcal{C}$ is the set of weighted prime implicants of B_1 and $\mathcal{D}$ is the set of weighted prime implicants of B_2, then the degree of conflict between B_1 and B_2 is defined as*

$$D_{Con}(B_1, B_2) = min\{d_{Con}(C, D) | C \in \mathcal{C},\ D \in \mathcal{D}\}. \tag{11}$$

The degree of conflict measures to what extent B_1 and B_2 are in conflict with each other.

Let us look at an example to illustrate the advantage of our degree of conflict.

Example 7. *Let us consider the dialogue between three people John, Mary, and Gary, they are discussing "whether Italy is the best football team in the world "(p) and "whether the best forwards are in Brazil" (q). John says "I think Italy is the best football team in the world and the best forwards are in Brazil", Mary says "No, I think France is the best team, but I agree with you that the best forwards are in Brazil", and Gary says "No, I think France is the best team". Suppose all three people are certain of their beliefs. So the possibilistic knowledge bases are* $John = \{(p,1),(q,1)\}$*,* $Mary = \{(\neg p,1),(q,1)\}$ *and* $Gary = \{(\neg p,1)\}$*. To apply Equation 9, we consider the classical knowledge bases corresponding to these possibilistic knowledge bases. We then have* $C(John, Mary) = C(John, Gary) = 1/2$*. This is not reasonable, because John and Mary agree on q so the degree of conflict between them should be less than the degree of conflict between John and Gary. By contrast, we have* $D_{Con}(John, Mary) = 1/2$ *and* $D_{Con}(John, Gary) = 2/3$*, so* $D_{Con}(John, Mary) < D_{Con}(John, Gary)$*.*

The relationship between our degree of conflict and that defined by Hunter can be seen by the following proposition.

Proposition 11. *Let* B_1*,* B_2 *be two classical knowledge bases. Suppose* $C(B_1, B_2)$ *and* $D_{Con}(B_1, B_2)$ *are degrees of conflict defined by Definition 11 and Definition 13 respectively. Then* $C(B_1, B_2) \leq D_{Con}(B_1, B_2)$*.*

The following example shows that the degree of conflict will not always increase or decrease when one of them becomes logically stronger.

Example 8. *Let* $B_1 = \{(p, 0.8), (q, 0.6)\}$ *and* $B_2 = \{(\neg p, 0.5), (\neg q, 0.7)\}$ *be two possibilistic knowledge bases. So the degree of conflict between* B_1 *and* B_2 *is* $D_{Con}(B_1, B_2) = 0.55$*. However, if* B_1 *is revised to* $B_1' = \{(p, 0.8), (q, 0.6), (r, 0.4)\}$*, then* $D_{Con}(B_1', B_2) = 0.5$*. If* B_1 *is revised to* $B_1'' = \{(p, 0.8), (q, 0.6), (r, 0.9)\}$ *and* B_2 *is revised to* $B_2'' = \{(\neg p, 0.5), (\neg q, 0.7), (\neg r, 0.9)\}$*, then* $D_{Con}(B_1'', B_2'') = 0.67$*.*

Two knowledge bases are not in conflict if and only if their degree of conflict is 0.

Proposition 12. *Let* B_1 *and* B_2 *be two possibilistic knowledge bases. Then* $B_1 \cup B_2$ *is consistent if and only if* $D_{Con}(B_1, B_2) = 0$*.*

The proof of Proposition 12 is similar to that of Proposition 5.

Similarly, we can define the degree of strong agreement between two possibilistic knowledge bases. It is easy to see that the influences of the quantity of conflict on the degree of strong agreement are more than that of the quantity of weak agreement.

Definition 14. *Let B_1 and B_2 be two possibilistic knowledge bases. Let C be a weighted prime implicant of B_1 and D be a weighted prime implicant of B_2. $Atom_{SA}(C, D)$ denotes the cardinality of the set of atoms which are included in both C and D. Then the degree of strong agreement between C and D is defined as*

$$d_{SA}(C, D) = \frac{q_{SA}(C, D)}{Atom_{SA}(C, D) + q_{Con}(C, D) + \lambda q_{WA}(C, D)}, \quad (12)$$

where $\lambda \in (0, 1]$ is used to weaken the influence of the quantity of weak agreement on the degree of strong agreement. As in Definition 12, we usually take $\lambda = 0.5$.

Definition 15. *Let B_1 and B_2 be two possibilistic knowledge bases. Suppose $\mathcal{C}$ is the set of weighted prime implicants of B_1 and $\mathcal{D}$ is the set of weighted prime implicants of B_2, then the degree of strong agreement between B_1 and B_2 is defined as*

$$D_{SA}(B_1, B_2) = max\{d_{SA}(C, D)|C\in\mathcal{C},\ D \in \mathcal{D}\}. \quad (13)$$

The degree of strong agreement measures to what extent B_1 and B_2 are in strong agreement with each other. Dual to the degree of conflict, we use the maximum to aggregate the degrees of strong agreement between two weighted prime implicants in $\mathcal{C}$ and weighted prime implicants in $\mathcal{D}$.

Example 9. *Let $B_1 = \{(p, 0.8), (q \vee r, 0.4), (p \to s, 0.5)\}$ and $B_2 = \{(p \vee \neg r, 0.8), (q,\ 0.6), (\neg s, 0.7)\}$. The weighted prime implicants of B_1 are $C_1 = \{(p, 0.8), (q, 0.4), (s, 0.5)\}$ and $C_2 = \{(p, 0.8), (r, 0.4), (s, 0.5)\}$, and the weighted prime implicants of B_2 are $D_1 = \{(p, 0.8), (q, 0.6), (\neg s, 0.7)\}$ and $D_2 = \{(\neg r, 0.8), (q, 0.6), (\neg s, 0.7)\}$. So $d_{SA}(C_1, D_1) = 0.48$, $d_{SA}(C_1, D_2) = 0.17$, $d_{SA}(C_2, D_1) = 0.4$, $d_{SA}(C_2, D_2) = 0$. Therefore, $D_{SA}(B_1, B_2) = 0.48$.*

The following example shows that the degree of strong agreement between two possibilistic knowledge bases will not always increase or decrease when one of them becomes logically stronger.

Example 10. *Let* $B_1 = \{(p, 0.8), (q, 0.9)\}$ *and* $B_2 = \{(p, 0.6), (q, 0.8)\}$*. Then we have* $D_{SA}(B_1, B_2) = 0.7$*. Suppose a new possibilistic formula* $(r, 0.8)$ *is added to* B_2 *and the new possibilistic knowledge base is denoted as* B_2'*, then we have* $D_{SA}(B_1, B_2') = 7/12$*. So* $D_{SA}(B_1, B_2') < D_{SA}(B_1, B_2)$*. However, if we remove* $(q, 0.8)$ *from* B_2 *and get* B_2''*, then we have* $D_{SA}(B_1, B_2'') = 12/29$*. Therefore* $D_{SA}(B_1, B_2) > D_{SA}(B_1, B_2'')$*.*

The degree of conflict and the degree of strong agreement are related with each other, which can be seen from the following proposition.

Proposition 13. *Let* B_1 *and* B_2 *be two possibilistic knowledge bases. If* $D_{Con}(B_1, B_2) > 0.5$*, then* $D_{SA}(B_1, B_2) < 0.5$ *and vice verse.*

Proposition 13 says that for any two possibilistic knowledge bases, their degree of conflict and degree of strong agreement cannot be greater than 0.5 at the same time.

We can also define the degree of weak agreement.

Definition 16. *Let* B_1 *and* B_2 *be two possibilistic knowledge bases. Let* C *be a weighted prime implicant of* B_1 *and* D *be a weighted prime implicant of* B_2*.* $Atom_{WA}(C, D)$ *denotes the cardinality of the set of atoms which are included in only one of* C *and* D *but not both. Then the degree of weak agreement between* C *and* D *is defined as*

$$d_{WA}(C, D) = \frac{q_{WA}(C, D)}{Atom_{WA}(C, D) + q_{Con}(C, D) + q_{SA}(C, D)}, \tag{14}$$

In Definition 16, the quantity of conflict and quantity of strong agreement have the same influence on the degree of weak agreement. When both B_1 and B_2 are classical knowledge bases, then we have $d_{WA}(C, D) = \frac{Atom_{WA}(C,D)}{|Atom(C \cup D)|}$.

Definition 17. *Let* B_1 *and* B_2 *be two possibilistic knowledge bases. Suppose* $\mathcal{C}$ *is the set of weighted prime implicants of* B_1 *and* $\mathcal{D}$ *is the set of weighted prime implicants of* B_2*, then the degree of weak agreement between* B_1 *and* B_2 *is defined as*

$$D_{WA}(B_1, B_2) = max\{d_{WA}(C, D) | C \in \mathcal{C},\ D \in \mathcal{D}\}. \tag{15}$$

The degree of weak agreement measures to what extent B_1 and B_2 are in weak agreement with each other.

Example 11. *(Continuing Example 9) By Definition 16, we have* $d_{WA}(C_1, D_1) = 0$*,* $d_{WA}(C_1, D_2) = 0.55$*,* $d_{WA}(C_2, D_1) = 0.3$*,* $d_{WA}(C_2, D_2) = 0.48$*. So the degree of weak agreement between* B_1 *and* B_2 *is 0.55.*

The following example shows that the degree of weak agreement between two possibilistic knowledge bases will not always increase or decrease when one of them becomes logically stronger.

Example 12. *(Continue Example 10) It is easy to check that* $D_{WA}(B_1, B_2) = 0$*,* $D_{WA}(B_1, B_2') = 8/27$ *and* $D_{WA}(B_1, B_2'') = 9/16$*. So* $D_{SA}(B_1, B_2) < D_{SA}(B_1, B_2)$ *but* $D_{SA}(B_1, B_2'') > D_{SA}(B_1, B_2)$*.*

It is possible that the degree of conflict and the degrees of agreement between two knowledge bases are less than 0.5 at the same time. For example, let $B_1 = \{p, q, r)\}$ and $B_2 = \{p, \neg q\}$, then $D_{Con}(B_1, B_2) = D_{SA}(B_1, B_2) = 0.4$ and $D_{WA}(B_1, B_2) = 1/3$. It is also possible that the degree of conflict between two possibilistic knowledge bases is greater than $1/2$ and the degree of weak agreement between them is also greater than $1/2$. For example, let $B_1 = \{(p, 1), (q, 0.9), (r, 0.9)\}$ and $B_2 = \{(\neg p, 1)\}$, then we have $D_{Con}(B_1, B_2) = 1/1.9$ and $D_{WA}(B_1, B_2) = 1.8/2.8$. Therefore, both $D_{Con}(B_1, B_2)$ and $D_{WA}(B_1, B_2)$ are greater than $1/2$.

It is easy to check that functions D_{Con}, D_{SA} and D_{WA} are all syntax-independent.

Proposition 14. *Given four possibilistic knowledge bases* B_1*,* B_1'*,* B_2 *and* B_2'*, suppose* $B_1 \equiv_s B_1'$ *and* $B_2 \equiv_s B_2'$*, then* $D(B_1, B_2) = D_{Con}(B_1', B_2')$*, where* D *is either* D_{Con} *or* D_{SA} *or* D_{WA}*.*

5. Impact of Measures of Conflict and Agreement

In this section, we discuss some applications of the measures of conflict and agreement. We first consider how the degree of conflict and degree of strong agreement can be used to guide the choice of combination operators in possibilistic logic. We then apply the degree of conflict to define an ordering relation between two possibilistic knowledge bases.

5.1. Choice of combination operators

Many combination rules in possibilistic logic have been proposed [3, 2]. Let B_1 and B_2 be two possibilistic knowledge bases, π_1 and π_2 be their associated possibility distributions. Semantically, a two place function $\oplus$ from [0,1]×[0,1] to [0,1], is applied to aggregate π_1 and π_2 into a new possibility distribution $\pi_\oplus$, i.e. $\pi_\oplus(\omega) = \pi_1(\omega) \oplus \pi_2(\omega)$. Generally, the operator $\oplus$ is very weakly constrained, i.e. the only requirements for it are the following properties [2]:

1. $1 \oplus 1 = 1$, and
2. if $a \geq c$, $b \geq d$ then $a \oplus b \geq c \oplus d$, where $a, b, c, d \in [0, 1]$ (monotonicity).

Two basic operators are the maximum and the minimum. Given two possibility distributions π_1 and π_2, let $\pi_{dm}(\omega) = max(\pi_1(\omega), \pi_2(\omega))$ and $\pi_{cm}(\omega) = min(\pi_1(\omega), \pi_2(\omega))$. The merging operators based on the maximum and the minimum have no reinforcement effect. That is, given an interpretation ω, if expert 1 assigns possibility $\pi_1(\omega) < 1$ and expert 2 assigns possibility $\pi_2(\omega) < 1$ to ω, then overall $\pi_{dm}(\omega) = \pi_2(\omega)$ (or $\pi_{cm}(\omega) = \pi_1(\omega)$) if $\pi_1(\omega) < \pi_2(\omega)$, regardless of the value of $\pi_1(\omega)$ (or $\pi_2(\omega)$). To obtain a reinforcement effect, we can use a triangular norm operator other than the minimum for conjunctive combination, and a triangular conorm operator other than the maximum for disjunctive combination.

Definition 18. *A triangular norm (t-norm) tn is a two place real-valued function* $tn : [0, 1] \times [0, 1] \rightarrow [0, 1]$ *which satisfies the following conditions:*

1. *tn(0,0)=0, and tn(α,1)=tn(1,α)=α, for every α (boundary condition);*
2. *tn(α_1,α_2)$\leq$tn(β_1,β_2) whenever $\alpha_1 \leq \beta_1$ and $\alpha_2 \leq \beta_2$ (monotonicity);*
3. *tn(α,β)=tn(β,α) (symmetry);*
4. *tn(α,tn(β,γ))=tn(tn(α,β),γ) (associativity).*

A triangular conorm (t-conorm) is a two place real-valued function $ct : [0, 1] \times [0, 1] \rightarrow [0, 1]$ *which satisfies the conditions 2-4 given in Definition 18 plus the following revised boundary conditions:*

1'. *ct(1,1)=1,ct(α,0)=ct(0,α)=α.*

Any t-conorm *ct* can be generated from a t-norm through the duality transformation:

$$ct(\alpha, \beta) = 1 - tn(1 - \alpha, 1 - \beta)$$

and conversely.

It is easy to check that the maximum operator is a t-conorm and the minimum operator is a t-norm. Other frequently used t-norms are the product operator $\alpha\beta$ and the *Lukasiewicz t-norm* $(max(0, \alpha + \beta - 1))$. The duality relation yields the following t-conorms respectively: the *probabilistic sum* $(\alpha + \beta - \alpha\beta)$, and the *bounded sum* $(min(1, \alpha + \beta))$.

Suppose two possibilistic knowledge bases B_1 and B_2 are jointly consistent, that is, $B_1 \cup B_2$ is consistent, then the degree of conflict between them must be 0 and at least one of the degrees of agreement is greater than 0. If the degree of strong agreement between B_1 and B_2 is very high, then B_1 and B_2 share beliefs on most of the topics. In this case, it is advisable to combine them using an operator with higher *reinforcement* effect, for example, the *Lukasiewicz t-norm* max(0,a+b-1). However, if the degree of strong agreement between B_1 and B_2 is low and the degree of weak agreement between them is very high, it is advisable to combine them using the minimum operator which does not have any *reinforcement* effect.

Suppose B_1 and B_2 are in conflict, we usually use a t-conorm to combine them. When the degree of conflict between B_1 and B_2 is very high, then B_1 and B_2 have mostly different beliefs and we can choose the "bounded sum" operator which has a high *counteract* effect. On the other hand, if the degree of conflict between B_1 and B_2 is very low, we can choose the maximum which does not have any *counteract* effect.

More formally, we have the following criteria to choose different t-norms (or t-conorms).

Merging operators selection criteria: Let $\oplus_1$ and $\oplus_2$ be two operators applied to merge A and B, and C and D respectively, then for all a, $b \in [0, 1]$,
(1) $\oplus_1(a, b) \leq \oplus_2 (a, b)$ if $0 < D_{Con}(A, B) < D_{Con}(C, D)$
(2) $\oplus_1(a, b) \geq \oplus_2 (a, b)$ if $D_{Con}(A, B) = 0$ and $D_{SA}(A, B) < D_{SA}(C, D)$.

More precisely, given a set of knowledge bases which are consistent together, i.e., their union is consistent, we have the following criteria to choose an operator by Equation (2) given above.

Definition 19. *Let the merging operators be the minimum operator min, the product operator $*$, and the Lukasiewicz t-norm $\oplus_L$, then we have the following criteria to choose an operator. Given n possibilistic knowledge bases B_1,...,B_n, suppose $B_1 \cup ... \cup B_n$ is consistent, let λ_1 and λ_2 be two real numbers such that $\lambda_1 < \lambda_2$,*

If $D_{SA}(B_1, ..., B_n) \geq \lambda_2$, the merging operator is the Lukasiewicz t-norm*;*

If $D_{SA}(B_1, ..., B_n) \in (\lambda_1, \lambda_2)$*, the merging operator is the product operator;*
If $D_{SA}(B_1, ..., B_n) \leq \lambda_1$*, the merging operator is the minimum operator.*

In Definition 19, λ_1 and λ_2 are thresholds that representing low and high degree of strong agreement respectively. For example, we may take λ_1 as 0.3 and λ_2 as 0.8. According to Definition 19, we choose the *Lukawiewicz t-norm* when the degree of strong agreement among B_i is high; we choose the minimum operator when the degree of strong agreement among them is low; and we choose the product operator otherwise. We have applied this selection criteria to define an adaptive algorithm to merge multiple possibilistic knowledge bases in [24].

Example 13. *Let* $B_1 = \{(p, 0.6), (q \vee \neg r, 0.7), (s, 0.6)\}$ *and* $B_2 = \{(p, 0.5), (q, 0.4), (s, 0.4)\}$*, where* $D_{Con}(B_1, B_2) = 0$ *and* $D_{SA}(B_1, B_2) = 0.43$*. The merging operator here should be the* product *operator, and the result of merging is* $B = \{(p, 0.6), (q\vee\neg r, 0.7), (s, 0.6), (p, 0.5), (q, 0.4), (s, 0.4), (p, 0.8), (p\vee q, 0.76), (p\vee s, 0.76), (p\vee q\vee\neg r, 0.85), (q\vee\neg r, 0.88), (q\vee\neg r\vee s, 0.88), (p\vee s, 0.8), (q \vee s, 0.76), (s, 0.76)\}$*. However, if we use the* Lukasiewicz t-norm*, the result of merging is* $B' = \{(p, 0.6), (q\vee\neg r, 0.7), (s, 0.6), (p, 0.5), (q, 0.4), (s, 0.4), (p, 1), (p\vee q, 1), (p\vee s, 1), (p\vee q\vee\neg r, 1), (q\vee\neg r, 1), (q\vee\neg r\vee s, 1), (p\vee s, 1), (q\vee s, 1), (s, 1) \}$*. In* B'*, the weights of formulas p and s are reinforced to 1. However, the certainty degrees of p and s are not high in both* B_1 *and* B_2*. Moreover,* B_1 *and* B_2 *are not in strong agreement with each other because* $D_{SA}(B_1, B_2) = 0.43$*. So it is not reasonable to increase the weights of p and s to the highest certainty degree 1. In contrast, in B, p and s have certainty degrees of 0.8 and 0.76 respectively. Therefore the result of the product operator reflects the reinforcement of* B_1 *and* B_2 *more accurately than that of the* Lukasiewicz t-norm*.*

5.2. Ordering sources

Measures of conflict and agreements can be used to rank knowledge bases. In [30], we defined a distance-based ordering between two knowledge bases in possibilistic logic based on the quantity of conflict, which is applied to define a belief negotiation model used to deal with the problem of belief merging. In [24], an ordering relation between two possibilistic knowledge bases in relation to a reference knowledge base is defined by using the degree of strong agreement, the inconsistency degree and the degree of conflict. This ordering relation is then used to define an adaptive merging operator. In this section, we define an ordering relation to compare different knowledge bases based on the degree of conflict.

Definition 20. *Let B_i, B_j, B be three possibilistic knowledge bases. A closeness relation $\preceq_B$ with regard to B is defined as.*

$$B_i \preceq_B B_j \ iff \ D_{Con}(B_j, B) \leq D_{Con}(B_i, B)$$

B_j is closer to B than B_i to B ($B_i \preceq_B B_j$) iff degree of conflict between B_j and B is less than or equal to that between B_i and B. If $B_i \preceq_B B_j$, then we may view B_j is less problematic or more reliable than B_i with regard to B. It is clear that the closeness relation is a total pre-order. Given a possibilistic knowledge base B, according to Proposition 5, any possibilistic knowledge base which is consistent with B is closest to it. When using the closeness relation, we may assume that the background knowledge base B is correct, then rank other knowledge bases by the degrees of conflict between them and B.

Example 14. *Let $B_1 = \{(\neg p, 0.8), (\neg q, 0.5), (\neg r \vee s, 0.7)\}$, $B_2 = \{(\neg p, 0.8), (\neg q, 0.5), (\neg r, 1), (s, 0.7)\}$, and $B = \{(p \vee q, 0.8), (\neg s \vee r, 1)\}$. Since $D_{Con}(B_1, B) = 0.22 < 0.41 = D_{Con}(B_2, B)$, so $B_2 \preceq_B B_1$.*

6. Related Work

We have discussed the relationship between our quantity of conflict with Dalal's distance between two knowledge bases and the relationship between our degree of conflict with the degree of conflict defined by Hunter in [18]. Our measures of conflict are also related to the measure of contradiction [23] or measures of inconsistency [21, 16, 22, 12, 19, 26]. The work in [12] extended some measures of information and conflict given in [16] and [17]. One may think that we can measure the degree of conflict between two knowledge bases by using measures of contradiction or measures of inconsistency of their union. More specifically, to measure the degree of conflict between two knowledge bases B_1 and B_2, we first concatenate them as $B_1 \cup B_2$, then take the degree of conflict between B_1 and B_2 as the degree of contradiction or degree of inconsistency [3]. However, this method is not de-

[3]When the knowledge bases are classical, we can use the measures defined on classical knowledge base given in [23, 19]. Whilst for possibilistic knowledge bases, we can apply the degree of inconsistency given in [12] or use the inconsistency degree defined in Section 2.2.

sirable because it fails to distinguish between B_1 and B_2. Consider three knowledge bases $K = \{(p, 0.6), (q, 0.8)\}$, $K_1 = \{(\neg p, 0.7), (r, 0.8)\}$ and $K_2 = \{(\neg p, 0.7), (q, 0.8), (r, 0.8)\}$. Since $K \cup K_1 = K \cup K_2 = \{(p, 0.6), (\neg p, 0.7), (q, 0.8), (r, 0.8)\}$, if we apply the aforementioned method, such as those given in [12], then the degree of conflict between K and K_1 is the same as that between K and K_2. This is counter-intuitive because there is more agreement between K and K_2 than there is between K and K_1.

In contrast to much work on measures of conflict between two knowledge bases and in a single knowledge base, there is relatively little work on measures of agreement between two knowledge bases. In [10], the authors define degrees of agreement between users and between users and information systems. Their degree of agreement for a pair of users is equal to the number of identical names chosen by two users divided by the number of names generated by each user, so it bears some similarity to our degree of strong agreement between two knowledge bases. One of the coherence measures given by Glass in [15] is closely related to our degrees of agreements. Suppose B_1 and B_2 are propositional knowledge bases. He first define the agreement between two classical interpretations ω_1 and ω_2 as $C(\omega_1, \omega_2) = \frac{|\omega_1 \cap \omega_2|}{|n|}$, where n is the number of atoms in the language and we treat interpretations as the set of literals that are true in them. He then define a coherence measure C between two knowledge bases B_1 and B_2 as follows:

$$C(B_1, B_2) = \frac{|M(B_1) \cap M(B_2)| + \delta}{|M(B_1) \cup M(B_2)|},$$

where $M(B_i)$ represents $M(B_i, B_1 \cup B_2)$ and

$$\delta = \frac{\Sigma_{\omega \in M_1} \Sigma_{\omega' \in M_2} C(\omega_1, \omega_2)}{|M(B_2)|} + \frac{\Sigma_{\omega \in M_1} \Sigma_{\omega' \in M_2} C(\omega_1, \omega_2)}{|M(B_1)|}$$

with $M_1 = M(B_1) \setminus M(B_2)$ and $M_2 = M(B_2) \setminus M(B_1)$.

For example, we have $C(\{p\}, \{p, q\}) = 3/4$, $C(\{p\}, \{p \vee q\}) = 3/4$, $C(\{\neg p\}, \{p, q\}) = 1/4$ and $C(\{p, q\}, \{\neg p, \neg q\}) = 0$. This measure does not differentiate between strong agreement and weak agreement between two knowledge bases. It can be considered as a measure of overall degree of agreement between two knowledge bases. There is no clear relationship between the degrees of strong agreement and weak agreement and degree of coherence. In the above example, we have $D_{SA}(\{p\}, \{p, q\}) = 2/3 < C(\{p\}, \{p, q\})$ and $D_{SA}(\{p\}, \{p \vee q\}) = 1 > C(\{p\}, \{p \vee q\})$, and $D_{WA}(\{p\}, \{p, q\}) = 1/2 < C(\{p\}, \{p, q\})$ and $D_{WA}(\{\neg p\}, \{p, q\}) = 1 > C(\{\neg p\}, \{p, q\})$.

A couple of similarity measures on possibility distributions have been given in the literature (see [20] for a survey). A set of natural properties of a similarity measure is given in [20] where they also defines a new similarity measure that satisfies all the properties. These measures can be adapted to measure similarity between two possibilistic knowledge bases. One may wonder if one of our degrees of agreement can be serve as a similarity measure. We show that none of our degree of agreement is a similarity measure in the sense that both of them do not satisfy a basic property adapted from a property for a similarity measure given in [20]. We first state this adapted property: If the range of a similarity measure s is the interval [0,1], then the upper bound of s is equal to 1. For any possibilistic knowledge bases B_1 and B_2, $s(B_1, B_2) = 1$ if and only if $B_1 \equiv_s B_2$. Our measure of strong agreement does not satisfy this property. Consider $B_1 = \{(p, 1)\}$ and $B_2 = \{(p \vee q, 1)\}$, we have $D_{SA}(B_1, B_2) = 1$ but B_1 and B_2 are not equivalent. Our measure of weak agreement does not satisfy this property as well. Consider $B_1 = \{(p, 1)\}$ and $B_2 = \{(q, 1)\}$, we have $D_{WA}(B_1, B_2) = 1$ but B_1 and B_2 are not equivalent.

7. Conclusion and Future Work

In this paper, we proposed measures of conflict between two prioritized knowledge bases and measures of agreement of two such bases. We defined the quantity of conflict and two quantities of agreement. The quantity of conflict is a generalization of the Dalal distance. We then defined the degree of conflict and degrees of agreement based on both the quantity of conflict and the quantities of agreement. We have shown that the definition of degree of conflict is more reasonable than that defined in [18]. The measures of conflict and agreement can be very useful in many applications, such as belief merging, argumentation and heterogeneous source integration and management. We can also apply measures of conflict to belief revision. Dalal in [11] proposed a revision operator which is defined by the so-called Dalal distance. As we have shown that the quantity of conflict generalizes the Dalal distance, it is possible to generalize Dalal's revision operator to possibilistic logic. There are several applications of the notion of a weighted prime implicant which generalizes the notion of a prime implicant. Prime implicants have been show useful in consequence finding (see [27]), in belief revision (see [7]) and in knowledge compilation (see [9]). Accordingly, we can apply weighted prime implicants to deal with similar problems in possibilistic logic.

When defining the measures of agreement and conflict, we assume that two possibilistic knowledge bases share a common scale. This assumption is also called the commensurability assumption. However, in practice, this assumption does not always hold (see [4] and [29]). As a future work, we will consider defining measures of agreement and conflict between two possibilistic knowledge bases that use different scales.

8. Proofs

Proof of Proposition 1

Suppose D is subsumed by D', then $D \neq D'$, and every literal appearing in D' must appear in D with higher or same necessity degree. Therefore, it is clear that $D \neq D'$ and $D \vdash_\pi D'$.

Conversely, suppose $D \neq D'$ and $D \vdash_\pi D'$. We only need to show that every literal appearing in D' must appear in D with higher or same necessity degree. Suppose (l, a) is in D'. Since $D \vdash_\pi D'$, we have $D \vdash_\pi (l, a)$. So $D_{\geq a} \vdash l$. It follows that $l \in D_{\geq a}$. Therefore, l appears in D with a necessity degree greater than or equal to a.

Proof of Proposition 2

A possibilistic knowledge base $D = \{(\psi_1, 1), ..., (\psi_k, 1)\}$, where ψ_j ($j = 1, k$) are literals, is a weighted implicant of B iff $D \vdash_\pi (\phi_i, 1)$ for all $(\phi_i, 1) \in B$ and there are no two complementary literals and $\psi_i \neq \psi_j$ for $i \neq j$. According to [13], $D \vdash_\pi (\phi_i, 1)$ iff $D \vdash \phi_i$ for all i. So D is a weighted implicant of B iff D is an implicant of B. According to the definition of subsumption between two weighted implicants, it is easy to check that D is a weighted prime implicant of B iff D is a prime implicant of B.

Proof of Proposition 3

"only if part": Suppose $B \vdash_\pi (\phi, a)$. Since $D_i \vdash_\pi B$, we have $D_i \vdash_\pi (\phi, a)$. Therefore $D_i \vdash_\pi (\phi, a)$, for all D_i.

"if part": Suppose $D_i \vdash_\pi (\phi, a)$ for all D_i, then $\vee_{i=1}^n (D_i)_{\geq a} \vdash \phi$. For each D_i, $D_i \vdash (\psi, b)$ for all $(\psi, b) \in B$ such that $b \geq a$. So $(D_i)_{\geq a} \vdash \psi$ for all $(\psi, b) \in B$ such that $b \geq a$. That is, $(D_i)_{\geq a}$ is an implicant of $B_{\geq a}$. For any prime implicant D of $B_{\geq a}$, there must exist some $(D_i)_{\geq a}$ such that $D \vdash (D_i)_{\geq a}$. Otherwise, we can construct a new weighted prime implicant of B from D. So $D \vdash \vee_{i=1}^n (D_i)_{\geq a}$. That is, $D \vdash \phi$. Therefore, we have $B_{\geq a} \vdash \phi$, i.e. $B \vdash_\pi (\phi, a)$.

Proof of Proposition 4

"only if part": Suppose D is a weighted prime implicant, we show the only if part by induction over b_i. We first prove that $D_{=b_1}$ is a prime implicant of $B_{=b_1}$ by absurdity. Suppose this does not hold, then there exists D_i which is a prime implicant of $B_{=b_1}$ such that $D_i \subset D_{b_1}$. So we can construct $D' = (D \setminus \{(l_i, b_1) : l_i \in D_{=b_1}\}) \cup \{(l_j, b_1) : l_j \in D_i\}$. It is easy to check that D' is a weighted implicant of B. However, $D' \neq D$ and $D \vdash_\pi D'$, contradiction. Assume that the only if part holds for b_i, where $i \leq k-1$. Similarly, we can show that $D_{=b_k}$ is a $D_{>b_k}$-extended prime implicant of $B_{=b_k}$ by absurdity.

"if part": Suppose it holds that $D_{=b_i}$ is a $D_{>b_i}$-extended prime implicant of $B_{=b_i}$ for $1 \leq i \leq k$, we show that D is a weighted prime implicant of B. It is easy to check that D is a weighted implicant of B. Suppose D is not a weighted prime implicant of B. Then there exists another different weighted implicant D' of B such that $D \vdash_\pi D'$. So there exists $i \in \{1, ..., k\}$ such that $D_{=b_j} = D'_{=b_j}$ for all $j < i$, but $D'_{=b_i} \subseteq D_{=b_i}$. This infers that $D_{=b_i}$ is not a $D_{>b_i}$-extended prime implicant of $B_{=b_i}$, contradiction.

Proof of Proposition 5

Suppose $B_1 \cup B_2$ is consistent, then there exist a model ω_1 of $(B_1)^*$ and a model ω_2 of $(B_2)^*$ such that $\omega_1 = \omega_2$ (both ω_1 and ω_2 should assign truth values to all the atoms appearing in $(B_1 \cup B_2)^*$). Suppose $Q_{Con}(B_1, B_2) \neq 0$. Then for any weighted prime implicant D_1 of B_1 and any weighted prime implicant D_2 of B_2, $q_{Con}(D_1, D_2) \neq 0$. According to Proposition 2, for any prime implicant $(D_1)^*$ of $(B_1)^*$ and any prime implicant $(D_2)^*$ of $(B_2)^*$, $q_{Con}((D_1)^*, (D_2)^*) \neq 0$. Since for any model ω of a classical knowledge base B, there exists a prime implicant D of B such that $\omega \models D$. If we consider $\omega = \{p \in PS | \omega(p) = true\} \cup \{\neg p | p \in PS, \omega(p) = false\}$, then we have $D \subseteq \omega$. Therefore, there does not exist a model ω_1 of $(B_1)^*$ and a model ω_2 of $(B_2)^*$ such that $\omega_1 = \omega_2$. This is a contradiction. So we must have $Q_{Con}(B_1, B_2) = 0$. Conversely, suppose $Q_{Con}(B_1, B_2) = 0$. Then there exist a weighted prime implicant D_1 of B_1 and a weighted prime implicant D_2 of B_2 such that $Q_{Con}(D_1, D_2) = 0$. By Proposition 2, $Q_{Con}((D_1)^*, (D_2)^*) = 0$. Therefore, we can find a model ω_1 of $(B_1)^*$ and a model ω_2 of $(B_2)^*$ such that $\omega_1 = \omega_2$. So B_1 and B_2 are not in conflict.

Proof of Proposition 6

We only need to prove that for any weighted prime implicant D of B and weighted prime implicant D_2 of B_2, there exists a weighted prime implicant D_1 of B_1 such that $q_{Con}(D, D_1) \leq q_{Con}(D, D_2)$. Since $B_2 \vdash_\pi B_1$ and

$D_2 \vdash_\pi B_2$, we have $D_2 \vdash_\pi B_1$. That is, D_2 is a weighted implicant of B_1. So there exists a weighted prime implicant D_1 of B_1 such that D_2 is subsumed by D_1. By Definition 5, we have $q_{Con}(D, D_1) \leq q_{Con}(D, D_2)$. Therefore, $Q_{Con}(B, B_1) \leq Q_{Con}(B, B_2)$.

Proof of Proposition 8

Without loss of generality, we suppose that the formulas in B_1 and B_2 are clauses.

We first prove $Dalal(B_1, B_2) \leq Q_{Con}(B_1, B_2)$. Let C and D be prime implicants of B_1 and B_2 respectively. Suppose w_1 and w_2 are two interpretations such that $w_1 \models p$ ($w_2 \models p$) if $p \in C$ ($p \in D$) and $w_1 \models \neg p$ ($w_2 \models \neg p$) if $\neg p \in C$ ($\neg p \in D$) and $w_1 \models p$ ($w_2 \models p$) otherwise. We then have $Dalal(\omega_1, \omega_2) = Q_{Con}(C, D)$. Moreover, $w_1 \models C$ and $w_2 \models D$ and $Dalal(w_1, w_2) = q_{Con}(C, D)$. Since $C \models B_1$ and $D \models B_2$, $w_1 \models B_1$ and $w_2 \models B_2$. So $w_1 \in M(B_1, B_1 \cup B_2)$ and $w_2 \in M(B_2, B_1 \cup B_2)$. By the definition of $Q_{Con}(B_1, B_2)$, we have $Dalal(B_1, B_2) \leq Q_{Con}(B_1, B_2)$.

Secondly, suppose $w_1 \in M(B_1, B_1 \cup B_2)$ and $w_2 \in M(B_2, B_1 \cup B_2)$. Then $w_1 \models B_1$ and $w_2 \models B_2$. Since $B_1 = C_1 \vee ... \vee C_n$, where C_i are the set of prime implicants of B_1, there must exist a prime implicant C_i of B_1 such that $w_1 \models C_i$. Similarly, there must exist a prime implicant D_j of B_2 such that $w_2 \models D_j$. So $w_1 \models p$ ($w_2 \models p$) if $p \in C$ ($p \in D$) and $w_1 \models \neg p$ ($w_2 \models \neg p$) if $\neg p \in C$ ($\neg p \in D$). So $q_{Con}(C, D) \leq Dalal(w_1, w_2)$. Therefore, $Q_{Con}(B_1, B_2) \leq Dalal(B_1, B_2)$.

Therefore, we must have $Q_{Con}(B_1, B_2) = Dalal(B_1, B_2)$.

Proof of Proposition 11

For any prime implicant C of B_1 and any prime implicant D of B_2, there exist $\omega_1 \in M(B_1, B_1 \cup B_2)$ and $\omega_2 \in M(B_2, B_1 \cup B_2)$ such that $q_{Con}(C, D) = Dalal(\omega_1, \omega_2)$. Since $Atom_{Con}(C, D) \leq log_2(|I(B_1 \cup B_2)|)$, we have that $\frac{Dalal(\omega_1, \omega_2)}{log_2(|I(B_1 \cup B_2)|)} \leq d_{Con}(C, D)$. So $C(B_1, B_2) \leq D_{Con}(B_1, B_2)$.

Proof of Proposition 13

Suppose $D_{Con}(B_1, B_2) > 0.5$, then for any weighted prime implicant C of B_1 and any weighted prime implicant D of B_2, $d_{Con}(C, D) > 0.5$. That is, $2q_{Con}(C, D) > Atom_{Con}(C, D) + q_{SA}(C, D) + \lambda q_{WA}(C, D)$. Suppose $D_{SA}(B_1, B_2) > 0.5$, then we have $2q_{SA}(C, D) > Atom_{SA}(C, D) + q_{Con}(C, D) + \lambda q_{WA}(C, D)$. We then have $q_{Con}(C, D) + q_{SA}(C, D) > Atom_{Con}(C, D) + Atom_{SA}(C, D) + 2\lambda q_{WA}(C, D)$. However, $q_{Con}(C, D) < Atom_{Con}(C, D)$ and $q_{SA}(C, D) < Atom_{SA}(C, D)$. This is a contradiction. Therefore, $D_{Con}(B_1, B_2)$ and $D_{SA}(B_1, B_2)$ cannot be great than 0.5 at the same time.

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