Combining Multiple Knowledge Bases by Negotiation: A Possibilistic Approach

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Abstract. A negotiation model consists of two functions: a negotiation function and a weakening function. A negotiation function is defined to choose the weakest sources and these sources will weaken their point of view using a weakening function. However, the currently available belief negotiation models are based on classical logic, which make it difficult to define weakening functions. In this paper, we define a prioritized belief negotiation model in the framework of possibilistic logic. The priority between formulae provides us with important information to decide which beliefs should be discarded. The problem of merging uncertain information from different sources is then solved by two steps. First, beliefs in the original knowledge bases will be weakened to resolve inconsistencies among them. This step is based on a prioritized belief negotiation model. Second, the knowledge bases obtained by the first step are combined using a *conjunctive* operator or a *reinforcement* operator in possbilistic logic.

1 Introduction

In recent years, some belief merging methods based on belief negotiation models were proposed to make the merging process more "active" [6, 7, 12]. Belief negotiation models based methods deal with the merging problem by several rounds of negotiation or competition. In each round, some sources are chosen by a negotiation function, then these sources have to weaken their point of view using a weakening function. However, both Konieczny's belief negotiation model and Booth's belief negotiation model are defined in purely propositional logic systems. So it is difficult for them to define a weakening function.

The importance of priorities in handling inconsistencies has been addressed by many researchers in recent years, e.g. [3, 11, 13]. Priority between formulae provides us with important information to decide which formulae should be discarded. So it is helpful to consider priority when we define a belief negotiation model. Possibilistic logic [9] provides a good framework to express priorities and reason with uncertain information. In possibilistic logic, each classical first order formula is attached with a number or weight, denoting the *necessity degree* of the formula. The necessity degrees can be interpreted as the priorities of formulae.

In this paper, we propose a prioritized belief negotiation model, where priorities between formulae are handled in the framework of possibilistic logic. Each source of beliefs is represented as a *possibilistic belief base*. The procedure of merging different sources of beliefs is carried out in two steps. The first step is called a negotiation step, beliefs in some of the original knowledge bases will be weakened to make it possible for them to be added together consistently (this step is called "social contraction" in [7]). Some negotiation functions and weakening functions will be defined by considering the priority in this step. The second step is called a combination step, the knowledge bases obtained by the first step are combined using a *conjunctive* operator or a *reinforcement* operator in possbilistic logic [2, 4].

This paper is organized as follows. We introduce Konieczny's belief game model in Section 2. Then in Section 3, we give a brief review of possibilistic logic. Our prioritized belief negotiation model will be presented in Section 4. In Section 5, we give some particular negotiation functions and weakening functions. In Section 6, we instantiate the prioritized belief negotiation model and provide an example to illustrate the new merging methods. Finally, we conclude the paper in Section 7.

2 Preliminaries

In this paper, we will consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$. $\mathcal{W}$ denotes the set of possible worlds, where each possible world is a function from $\mathcal{P}$ to $\{\top, \bot\}$ ($\top$ denotes truth value *true* and $\bot$ denotes the truth value *false*). A model of a formula ϕ is a possible world w which makes the formula *true*. We use $mod(\phi)$ to denote the set of models of formula ϕ, i.e., $mod(\phi) = \{w{\in}\mathcal{W}|w{\models}\phi\}$. Deduction in classical propositional logic is denoted by symbol $\vdash$ as usual. ϕ, ψ, γ,... represent classical formulae. Given two formulae ϕ and ψ, ϕ and ψ is equivalent, denoted as $\phi \equiv \psi$, if and only if $\phi \vdash \psi$ and $\psi \vdash \phi$.

A belief base φ is a consistent propositional formula (or, equivalently, a finite consistent set of propositional formulae). Let φ_1,...,φ_n be n belief bases (not necessarily different). A *belief profile* is a multi-set Ψ consisting of those n belief bases: $\Psi = (\varphi_1, ..., \varphi_n)$. The *conjunction* of the belief bases of Ψ is denoted as $\bigwedge\Psi$, i.e., $\bigwedge\Psi = \varphi_1\wedge...\wedge\varphi_n$. $\bigsqcup$ and $\sqsubseteq$ are used to denote the *union* and *inclusion* of belief profiles respectively. Two belief profiles Ψ_1 and Ψ_2 are said to be equivalent ($\Psi_1{\equiv}\Psi_2$) if and only if there is a bijection f between Ψ_1 and Ψ_2 such that $\forall\varphi{\in}\Psi_1$, $\varphi{\equiv}f(\varphi)$, where $f(\varphi)$ is the image of φ in Ψ_2. $\mathcal{E}$ denotes the set of all finite non-empty belief profiles.

Belief game model: A belief game model [12] is developed from Booth's belief negotiation model [7] which provides a framework for merging sources of beliefs incrementally. It consists of two functions. One is called a negotiation function, which selects from every belief profile in $\mathcal{E}$ a subset of belief bases. The other is called a weakening function, which aims to weaken the beliefs of a selected source.

Definition 1. *A* negotiation function *is a function $g : \mathcal{E} \rightarrow \mathcal{E}$ such that:*

$$(n1) \quad g(\Psi) \sqsubseteq \Psi, \quad (n2) \quad g(\Psi) \neq \emptyset, \quad (n3) \quad \exists \varphi \in g(\Psi) \quad s.t. \quad \varphi \not\equiv \top,$$
$$(n4) \quad If \quad \Psi \equiv \Psi', \quad then \quad g(\Psi) \equiv g(\Psi')$$

The first two conditions guarantee a non-empty subset is chosen from a belief profile to be weakened. The third condition states that at least one non-tautological base must be selected. The last condition is about irrelevance of syntax.

Definition 2. *A* weakening function *is a function $\nabla : \mathcal{L} \rightarrow \mathcal{L}$ such that:*

$$(w1)\ \varphi \vdash \nabla(\varphi),\ (w2)\ If\ \varphi \equiv \nabla(\varphi),\ then\ \varphi \equiv \top,$$
$$(w3)\ If\ \varphi \equiv \varphi',\ then\ \nabla(\varphi) \equiv \nabla(\varphi')$$

The first two conditions ensure that a base will be replaced by a strictly weaker one unless the base is already a tautological one. The last condition is an irrelevance of syntax requirement, i.e., the result of weakening depends only on the information conveyed by a base, not on its syntactical form.

A weakening function can be extended as follows. Let Ψ' be a subset of Ψ, $\nabla_{\Psi'}(\Psi) = \cup_{\varphi \in \Psi'} \nabla(\varphi) \cup \cup_{\varphi \in \Psi \setminus \Psi'} \varphi$.

Definition 3. *A Belief Game Model (BGM) is a pair $\mathcal{N} = \langle g, \nabla \rangle$ where g is a negotiation function and ∇ is a weakening function. The solution to a belief profile Ψ for a Belief Game Model $\mathcal{N} = \langle g, \nabla \rangle$, noted as $\mathcal{N}(\Psi)$, is the belief profile $\Psi_{\mathcal{N}}$, defined as:*

- $\Psi_0 = \Psi$
- $\Psi_{i+1} = \nabla_{g(\Psi_i)}(\Psi_i)$
- $\Psi_{\mathcal{N}}$ *is the first* Ψ_i *that is consistent*

3 Possibilistic Logic

Possibilistic logic [9] is a weighted logic where each classical logic formula is associated with a level of priority. A possibilistic belief base (**PBB**) is a set of possibilistic formulae of the form $B = \{(\phi_i,\ \alpha_i) : i = 1, ..., n\}$, where $\alpha_i \in [0, 1]$ and they are meant to be the necessity degrees of the ϕ_i. The classical base associated with B is denoted as B^*, namely $B^* = \{\phi_i | (\phi_i,\ \alpha_i) \in B\}$. A PBB B is consistent if and only if its classical base B^* is consistent. In possibilistic logic, a *possibility distribution*, denoted by π, is a mapping from a set of possible worlds $\mathcal{W}$ to the interval [0,1]. $\pi(\omega)$ represents the possibility degree of the interpretation ω with regard to available beliefs. From a *possibility distribution* π, two measures defined on a set of propositional or first order formulae can be determined. One is the possibility degree of formula ϕ, denoted as $\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$. The other is the necessity degree of formula ϕ, and is defined as $N(\phi) = 1 - \Pi(\neg\phi)$.

A possibilistic belief profile $\mathcal{KP}$ is a multi-set of PBBs, where these PBBs are not necessarily different. $\mathcal{KP} = (B_1, ..., B_n)$ is consistent iff $B_1^* \cup ... \cup B_n^*$ is consistent. We use $\mathcal{PE}$ to denote the set of all finite non-empty possibilistic belief profiles and $\mathcal{K}$ to denote the set of all the PBBs.

Definition 4. *Let B be a PBB, and $\alpha \in [0,1]$. The* α-cut *of B is* $B_{\geq\alpha} = \{\phi{\in}B^* | (\phi, a){\in}B \textit{ and } a{\geq}\alpha\}$.

The *inconsistency degree* of B, which defines its level of inconsistency, is defined as: $Inc(B) = max\{\alpha_i | B_{\geq\alpha_i} \textit{ is inconsistent}\}$.

Let B and B' be two PBBs. B and B' are said to be equivalent, denoted by $B \equiv_s B'$, iff $\forall\ a \in [0,1],\ B_{\geq a}{\equiv}B'_{\geq a}$. Two possibilistic belief profiles $\mathcal{KP}_1$ and $\mathcal{KP}_2$ are said to be equivalent ($\mathcal{KP}_1{\equiv_s}\mathcal{KP}_2$) if and only if there is a bijection between them such that each PBB of $\mathcal{KP}_1$ is equivalent to its image in $\mathcal{KP}_2$.

Definition 5. *Let B be a PBB. Let $(\phi,\ \alpha)$ be a piece of information with $\alpha{>}Inc(B)$. $(\phi,\ \alpha)$ is said to be a consequence of B, denoted by $B \vdash_\pi (\phi,\ \alpha)$, iff $B_{\geq\alpha} \vdash \phi$.*

Given a PBB B, a unique *possibility distribution*, denoted by π_B, can be obtained by the principle of minimum specificity. For all $\omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall(\phi_i,\ \alpha_i) \in B, \omega \models \phi_i, \\ 1 - max\{\alpha_i | \omega \not\models \phi_i, (\phi_i, \alpha_i) \in B\} & \text{otherwise.} \end{cases} \quad (1)$$

Many combination rules for merging PBBs have been proposed [2, 4]. Let us first introduce some operators which combine possibility distributions.

Definition 6. *[4] A conjunctive operator is a two place function $\oplus :\ [0,1] \times [0,1]{\rightarrow}[0,1]$ such that $\forall a{\in}[0,1]$, $a{\oplus}1 = 1{\oplus}a = a$.*

Examples of conjunctive operators the minimum operator and the product operator.

Definition 7. *[4] A reinforcement operator is a two place function $\oplus :\ [0,1] \times [0,1]{\rightarrow}[0,1]$ such that $\forall a, b{\neq}1$ and $a, b{\neq}0$, $a{\oplus}b{<}min(a, b)$.*

Examples of reinforcement operator are the product operator and the *Lukasiewicz t-norm* $max(0, a{+}b{-}1)$. It is clear a conjunctive operator may be a reinforcement operator.

Given two PBBs B_1 and B_2, and a conjunctive operator or a reinforcement operator $\oplus$, a semantic combination rule combines the possibility distributions π_{B_1} and π_{B_2} using $\oplus$ as $\pi_\oplus(w) = \pi_{B_1}(w){\oplus}\pi_{B_2}(w)$. Its syntactical counterpart is the following PBB [4]:

$$\begin{aligned} &B_1{\oplus}B_2 = \{(\phi_i, 1 - (1 - a_i){\oplus}1) : (\phi_i, a_i){\in}B_1\}{\cup}\{(\psi_j, 1 - 1{\oplus}(1 - b_j)) : (\psi_j, b_j) \\ &{\in}B_2\}{\cup}\{(\phi_i \vee \psi_j, 1 - (1 - a_i){\oplus}(1 - b_j)) : (\phi_i, a_i){\in}B_1 \textit{ and } (\psi_j, b_j){\in}B_2\}. \end{aligned} \quad (2)$$

For example, when $\oplus = min$, $B_1{\oplus}B_2 = B_1{\cup}B_2$. It is often assumed that an operator used to combine possibility distributions should be both commutative and associative, i.e., $a{\oplus}b = b{\oplus}a$ and $a{\oplus}(b{\oplus}c) = (a{\oplus}b){\oplus}c$. In this case, the order of the combination will not influence the result of merging when multiple PBBs need to be combined. When the union of original PBBs is consistent, it is advisable to use a conjunctive operator based combination rule or a reinforcement operator based combination rule because all the formulae in these PBBs are kept in the resulting PBB and their necessity degrees are not decreased.

4 A Prioritized Belief Negotiation Model

In this section, we will propose a prioritized belief negotiation model to generalize the belief game model [12], where priorities between formulae are handled in the framework of possibilistic logic. Each source of beliefs is represented as a PBB. We assume that the original PBBs are self-consistent.

Definition 8. *A* negotiation function *is a function g:* $\mathcal{PE} \rightarrow \mathcal{PE}$ *such that:*

$$(N1)\ g(\mathcal{KP}) \sqsubseteq \mathcal{KP}, \quad (N2)\ g(\mathcal{KP}) \neq \emptyset,$$

$$(N3)\ If\ \exists B \in \mathcal{KP}\ s.t.\ B^* \not\equiv \top,\ then\ \forall B' \in g(\mathcal{KP}),\ (B^*)' \not\equiv \top.$$

Conditions $N1$ and $N2$ are directly generalized from Conditions $n1$ and $n2$ in BGM. Condition N3 states that the negotiation function will not select the PBB whose classical base is equivalent to the "tautology" if there is a PBB whose classical base is not equivalent to the "tautology". That is, we do not choose the "tautology" to weaken if possible. Our negotiation function relies on the syntactical form of the PBBs, because every formula is attached a weight in a PBB, and we need to consider the syntax of the PBB.

Next we will give the definition of a weakening function.

Definition 9. *A* weakening function *is a function* ∇*:* $\mathcal{K} \times \mathcal{PE} \times \mathcal{PE} \rightarrow \mathcal{K}$ *such that: for each triple consisting of a PBB B and two possibilistic profiles* $\mathcal{KP}$ *and* $\mathcal{KP}'$*, if* $\mathcal{KP}' \sqsubseteq \mathcal{KP}$ *and* $B \in \mathcal{KP}'$*, then* $\nabla_{\mathcal{KP},\mathcal{KP}'}(B)$ *should satisfy the conditions (W1) and (W2) below, otherwise* $\nabla_{\mathcal{KP},\mathcal{KP}'}(B) = B$*.*

$$(W1)\ B \vdash_\pi (\phi, a),\ for\ all\ (\phi, a) \in \nabla_{\mathcal{KP},\mathcal{KP}'}(B)$$

$$(W2)\ If\ B = \nabla_{\mathcal{KP},\mathcal{KP}'}(B),\ then\ B^* \equiv \top$$

Unlike the weakening function in BGM, our weakening function only weakens the PBBs in a subset of possibilistic belief profile and keeps other PBBs unchanged. The priority between formulae in a PBB makes the construction of weakening function easy.

We can extend a weakening function on belief profiles as follows: let $\mathcal{KP}'$ be a subset of $\mathcal{KP}$, $\nabla_{\mathcal{KP},\mathcal{KP}'}(\mathcal{KP}) = \cup_{B \in \mathcal{KP}} \nabla_{\mathcal{KP},\mathcal{KP}'}(B)$.

Definition 10. *A prioritized belief negotiation model (relative to sources) is a pair* $\mathcal{N} =< g, \nabla >$ *where g is a negotiation function and* ∇ *is a weakening function. The solution to a possibilistic belief profile* $\mathcal{KP}$ *for a belief negotiation model* $\mathcal{N} =< g, \nabla >$*, noted as* $\mathcal{N}(\mathcal{KP})$*, is the belief profile* $\mathcal{KP}_\mathcal{N}$ *defined as:*

- $\mathcal{KP}_0 = \mathcal{KP}$
- $\mathcal{KP}_{i+1} = \nabla_{\mathcal{KP}_i, g(\mathcal{KP}_i)}(\mathcal{KP}_i)$
- $\mathcal{KP}_\mathcal{N}$ *is the first* $\mathcal{KP}_i$ *that is consistent.*

The merging of PBBs based on a prioritized belief negotiation model is to obtain a set of consistent PBBs through negotiation and then apply a suitable combination operator (usually a conjunctive operator or a reinforcement operator) to merge them.

5 Negotiation and Weakening Functions

5.1 Negotiation function

Distance-based negotiation function The first category of negotiation functions is based on a distance between two PBBs.

The following is the definition of a distance between two PBBs, which is a simple extension of the distance between two classical belief bases in [12].

Definition 11. *A (pseudo) distance between two PBBs is a function d: $\mathcal{KP} \times \mathcal{KP} \rightarrow [0,1]$ such that: $d(B,B')=0$ iff $B^* \cup B'^* \nvdash \bot$, and $d(B,B') = d(B',B)$.*

Clearly, a very simple distance can be defined as follows: $d_D(B,B') = 0$ if $B^* \cup B'^* \nvdash \bot$ and $d_D(B,B') = 1$ otherwise.

Now we will define a quantity of conflict between two PBBs based on *weighted prime implicants*. This can be used to define a distance between two PBBs.

An *implicant* of a belief base B is a conjunction of literals D such that $D \vdash B$ and D does not contain two complementary literals.

Definition 12. *A* prime implicant *of a belief base B is an implicant D of B such that for every other implicant D' of B, $D \nvdash D'$.*

Prime implicants are often used in knowledge compilation to make the deduction tractable. Suppose D_1, ..., D_k are all the prime implicants of B, we have $B \vdash \phi$ iff for every prime implicant D_i, $D_i \vdash \phi$, for any ϕ.

Now we define the weighted prime implicant of a PBB. Let us first define the weighted prime implicant for PBB $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ where ϕ_i are clauses, and a clause is a disjunction of literals. For a more general PBB, we can decompose it as an equivalent PBB whose formulae are clauses by the min-decomposability of necessity measures, i.e., $N(\wedge_{i=1,k}\phi_i) \geq m \Leftrightarrow \forall i, N(\phi_i) \geq m$ [10]. That is, a possibilistic formula $(\phi_1 \wedge ... \wedge \phi_k, a)$ can be equivalently decomposed as a set of possibilistic formulae (ϕ_1, a),...,(ϕ_k, a).

Let $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ be a PBB where ϕ_i are clauses. A weighted implicant of B is $D = \{(\psi_1, b_1), ..., (\psi_k, b_k)\}$, a PBB, such that $D \vdash_\pi B$, where ψ_i are literals. Let D and D' be two weighted implicants of B, D is said to be *subsumed* by D' iff $D \neq D'$, $D'^* \subseteq D^*$ and $\forall (\psi_i, a_i) \in D, \exists (\psi_i, b_i) \in D'$ with $b_i \leq a_i$ (b_i is 0 if $\psi_i \in D^*$ but $\psi_i \notin D'^*$).

Definition 13. *Let $B = \{(\phi_1, a_1), ..., (\phi_n, a_n)\}$ be a PBB where ϕ_i are clauses. A weighted prime implicant (***WPI***) of B is D such that*

1. *D is a weighted implicant of B*
2. *$\nexists$ D' of B such that D is subsumed by D'.*

Let us look at an example to illustrate how to construct WPIs.

Example 1. Let $B = \{(p, 0.8), (q \vee r, 0.5), (q \vee \neg s, 0.6)\}$ be a PBB. The WPIs of B are $D_1 = \{(p, 0.8), (q, 0.6)\}$, $D_2 = \{(p, 0.8), (r, 0.5), (\neg s, 0.6)\}$, and $D_3 = \{(p, 0.8), (q, 0.5), (\neg s, 0.6)\}$.

The WPI generalizes the prime implicant.

Proposition 1. *Let $B = \{(\phi_1, 1), ..., (\phi_n, 1)\}$ be a PBB where all the formulae have weight 1, i.e., B is a classical knowledge base. Then D is a WPI of B iff D is a prime implicant of B.*

However, given PBB B, if D is a WPI of B, then D^* is not necessary to be a prime implicant of B^*. A counterexample can be found in Example 1, where D_3 is a WPI, but $D_3^* = \{p, q, \neg s\}$ is not a prime implicant of B^*.

Definition 14. *Let B_1 and B_2 be two PBBs. Suppose C and D are WPIs of B_1 and B_2 respectively, then the quantity of conflict between C and D is defined as*

$$q_{Con}(C, D) = \Sigma_{(p,a)\in C \text{ and } (\neg p,b)\in D} min(a, b). \tag{3}$$

When the weights associated with all the formulae are 1, $q_{Con}(C, D)$ is the cardinality of the set of atoms which are in conflict in $C \cup D$.

Definition 15. *Let B_1 and B_2 be two PBBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of weighted prime implicants of B_1 and B_2 respectively, then the quantity of conflict between B_1 and B_2 is defined as*

$$Q_{Con}(B_1, B_2) = min\{q_{Con}(C, D) | C \in \mathcal{C},\ D \in \mathcal{D}\}. \tag{4}$$

The quantity of conflict between B_1 and B_2 measures information that is in conflict between B_1 and B_2. We have proved that the quantity of conflict between two classical belief bases are the Dalal distance between them [8] (We will not include the proof here due to the page limit.). So we can define a distance function d_C based on the quantity of conflict such that $d_C(B_1, B_2) = Q_{Con}(B_1, B_2)$ (it is easy to check that d_C satisfies the requirements of a distance function in Definition 11).

Definition 16. *[12] An aggregation function is a total function f associating a non-negative integer to every finite tuple of nonnegative integers and verifying the following conditions:*

- *if $x \leq y$, then $f(x_1, ..., x, ..., x_n) \leq f(x_1, ..., y, ..., x_n)$. (non-decreasingness)*
- *$f(x_1, ..., x_n) = 0$ iff $x_1 = ... = x_n = 0$. (minimality)*
- *for every nonnegative integer x, $f(x) = x$. (identity)*

Two most commonly used aggregation functions are the maximum and the sum Σ.

Now we can define the distance-based negotiation function.

Definition 17. *Let $\mathcal{KP} = \{B_1, ..., B_n\}$ be a multi-set of PBBs. A distance-based negotiation function is defined as follows: for all $B \in \mathcal{KP}$,*

$$B \in g^{d,f}(\mathcal{KP}) \quad iff \quad f(d(B, B_1), ..., d(B, B_n)) \text{ is maximal,}$$

where f is an aggregation function, d is a distance function between two PBBs.

Therefore, those sources that are "furthest" from the group are weakened.

Conflict-based negotiation function Priority provides an easy way for us to deal with inconsistency. In belief revision and belief merging, an implicit or explicit priority is often assumed. The inconsistency of a PBB can be resolved by dropping those formulae that are in conflict with lowest priorities in a minimally inconsistent subbase [5, 11]. A natural negotiation function can be defined by selecting those PBBs which contain conflict formulae in the lowest level of the union of all the PBBs.

Definition 18. *[3] A subbase C of PBB B is said to be minimally inconsistent if and only if it satisfies the following two requirements:* (1) $C^* \models \bot$, (2)$\forall \phi \in C^*,\ C^* - \{\phi\} \not\models \bot$.

Definition 19. *[3] A possibilistic formula* $(\phi,\ \alpha)$ *is said to be in* conflict *in B iff it belongs to some minimally inconsistent subbase of B.*

Definition 20. *Let B be an inconsistent PBB. A possibilistic formula (ϕ, a) is said to be a weakest conflict formula in B iff it satisfies* (1) *ϕ is in conflict in B,* (2) *$\forall (\psi, b) \in B$, if $b < a$, then ψ is not in conflict in B*

Definition 21. *Let $\mathcal{KP} = \{B_1, ..., B_n\}$ be a multi-set of PBBs. A weakest-conflict-based negotiation function is defined as follows:*

$$g^{wc}(\mathcal{KP}) = \{B_i \in \mathcal{KP} | \exists \text{ a weakest conflict formula in } \cup(\mathcal{KP}) \text{ belonging to } B_i\}.$$

The weakest-conflict-based negotiation function is often used with the weakest-conflict-based weakening function that will be defined in the next subsection.

5.2 Weakening function

The priority derived from the necessity degrees of possibilistic formulae allows us to define some syntax-based weakening functions. The first weakening function deletes the weakest conflict formulae in a belief base.

Definition 22. *Let B_1,...,B_n be PBBs and $\mathcal{KP} = \{B_1, ..., B_n\}$ be a possibilistic belief profile. A possibilistic formula (ϕ, a) is said to be the weakest conflict formula of B in $\mathcal{KP}$ iff*

- *ϕ is in conflict in $\cup(\mathcal{KP})$*
- *$\forall (\psi, b) \in B$, if $b < a$, then ψ is not in conflict in $\cup(\mathcal{KP})$*

Definition 23. *Let B_1,...,B_n be PBBs and $\mathcal{KP} = \{B_1, ..., B_n\}$ be a possibilistic belief profile and $\mathcal{KP}'$ be a subset of $\mathcal{KP}$. Let $B \in \mathcal{KP}'$ and $C = \{\phi \in B | \phi$ is a weakest $conflict\ formula\ of\ B\ in\ \cup(\mathcal{KP})\}$. The weakest-conflict-based (WC for short) weakening function is defined as:*

$$\bigtriangledown^{wc}_{\mathcal{KP},\mathcal{KP}'}(B) = B \backslash C.$$

The WC-weakening function deletes those formulae that are the weakest conflict formulae from a PBB which is selected by a negotiation function.

The weakening function defined above need to compute the conflict formulae, which is computationally too complex. In the following, we define a weakening function which does not need to compute conflict formulae.

Definition 24. *Let $\mathcal{KP} = \{B_1, ..., B_n\}$ be a possibilistic belief profile and $\mathcal{KP}'$ be an arbitrary subset of $\mathcal{KP}$. $B \in \mathcal{KP}'$. Let $\alpha = min\{a \in (0,1] : \exists \phi, (\phi, a) \in B\}$. The blind-optimized weakening function is defined as:*

$$\bigtriangledown^{bo}_{\mathcal{KP},\mathcal{KP}'}(B) = \{(\phi, a) \in B : a \neq \alpha\}.$$

The blind-optimized weakening function deletes formulae in the lowest layer. The weakening function applies when the agent does not know which formula is in conflict in the PBB, so it deletes those formulae that have the least priority.

6 Instantiating the Framework and Examples

6.1 Instantiation

Different combinations of the negotiation functions and the weakening functions will result in different prioritized belief negotiation models and then different belief merging methods. In the examples given below, we assume that after some PBBs are weakened, the combination operator is the minimum, i.e., the PBBs are conjoined.

- $\langle g^{wc}, \bigtriangledown^{wc} \rangle$[1]: This merging method deletes the conflict formulae from the lower levels, i.e weights of formulae are lower. That is, the agents always choose the weakest information to discard. This idea can be found in [5].
- $\langle g^{d_D, f^{Max}}, \bigtriangledown^{wc} \rangle$: In this case, every PBB which is in conflict with any of other PBBs deletes their weakest conflict formulae in each round. This merging method usually deletes more formulae than the merging method based on $\langle g^{wc}, \bigtriangledown^{wc} \rangle$.
- $\langle g^{d_D, f^{\Sigma}}, \bigtriangledown^{wc} \rangle$: In this case, in each round of negotiation, those PBBs which have the greatest number of PBBs in conflict will be selected and have their weakest conflict formulae deleted.
- $\langle g^{Q_{Con}, f^{\Sigma}}, \bigtriangledown^{wc} \rangle$: In this case, in each round of negotiation, those PBBs which have more quantities of information in conflict with other PBBs will be selected and have their weakest conflict formulae deleted.
- $\langle g^{Q_{Con}, f^{\Sigma}}, \bigtriangledown^{bo} \rangle$: In this case, in each round of negotiation, those PBBs which have more qunatities of information in conflict with other PBBs will be selected and have their lowest layers deleted. This merging method deletes more formulae than the merging method based on $\langle g^{Q_{Con}, f^{\Sigma}}, \bigtriangledown^{wc} \rangle$. However, it is computationally simpler.

[1] For simplicity, we will ignore the subscript of the weakening functions.

In the examples above, we require that the combination rule used in the second step of merging be the minimum. If we relax this restriction, we can get some more merging methods. For example, in the case of $\langle g^{wc}, \bigtriangledown^{wc}\rangle$, if we further assume that the combination operaotr is the product operator, then we can get a merging method which has a reinforcement effect.

Compared with merging methods in [1, 4], our methods are more *active*, i.e. agents resolve their conflicting information through the process of negotiation. Moreover, the merging results of our methods may retain more important information than those of methods in [1, 4]. For example, given two PBBs B_1 and B_2, a merging method in [1] first merges them using a *t-norm* operator through Equation 1, then deletes any formulae whose necessity degrees are under the inconsistency level of the resulting PBB. If the inconsistency degree of $B_1 \cup B_2$ is very high (0.9, for example), then possibilistic formulae in B_1 and B_2 whose necessity degrees are lower than 0.9 will be deleted even if some of them are not involved in conflict. However, using our methods, for example, the merging method which is based on the pair $\langle g^{Q_{Con}, f^{\Sigma}}, \bigtriangledown^{bo}\rangle$, some possibilistic formulae with necessity degrees lower than 0.9 can also be kept after merging.

6.2 Illustrative example

In this section, we will give an example to illustrate some prioritized belief negotiation model based merging methods, i.e., those based on $\langle g^{d_D, f^{\Sigma}}, \nabla^{wc}\rangle$ and $\langle g^{Q_C on, f^{\Sigma}}, \nabla^{wc}\rangle$.

Example 2. Three people are talking about origins of human beings and planets. Their opinions are summarized as weighted logical sentences in a possibilistic belief profile $\mathcal{KP} = \{A, B, C\}$, where

$$A = \{(p, 0.4), (q \rightarrow r, 1), (s, 0.8), (\neg s \rightarrow \neg r, 0.9)\}$$
$$B = \{(q, 0.8), (\neg s, 0.6), (e, 0.8)\}$$
$$C = \{(\neg p, 0.8), (\neg q, 0.6), (e \rightarrow r, 0.4)\}$$

- p represents "there were human beings in Mars before"
- q represents "scientists have detected some strange signals from outer space"
- r represents "there are aliens in other planets"
- s represents "the ancestors of human are gorillas"
- e represents "the earth was created by chance, not by a creator".

In this example, C is quite sure that there were no human beings in Mars before and is unsure that if the earth was created by chance, then there are aliens in other planets too.

Now we will see how they can negotiate with each other to make their opinions coherent.

- Method 1: $\langle g^{d_D, f^{\Sigma}}, \nabla^{wc}\rangle$ and $\oplus = Lukasiewicz\ t-norm$:
  Since A, B and C are in conflict, $g^{d_D, f^{\Sigma}}(\mathcal{KP}) = \mathcal{KP}$. So A is replaced by

$\nabla^{wc}(A) = \{(q \to r, 1), (s, 0.8), (\neg s \to \neg r, 0.9)\}$,[2] B is replaced by $\nabla^{wc}(B) = \{(q, 0.8), (e, 0.8)\}$ and C is replaced by $\nabla^{wc}(C) = \{(\neg p, 0.8), (\neg q, 0.6)\}$. Now $\nabla^{wc}(B)$ and $\nabla^{wc}(C)$ are still in conflict, and they will have to weaken their beliefs in the second round. So $\nabla^{wc}(B) = \{(e, 0.8)\}$ and $\nabla^{wc}(C) = \{(\neg p, 0.8)\}$. In this case, we have reached a consistent possibilistic belief profile. By combining $\nabla^{wc}(A)$, $\nabla^{wc}(B)$ and $\nabla^{wc}(C)$ using $Lukasiewicz\ t-norm$, we have the following result of merging:

$$\begin{aligned}\mathcal{KP}_{\oplus} = \{&(q \to r, 1), (s, 0.8), (\neg s \to \neg r, 0.9), (e, 0.8), (\neg p, 0.8), (e \vee \neg p, 1),\\ &(\neg q \vee r \vee e, 1), (s \vee e, 1), (\neg s \vee \neg r \vee e, 1), (\neg p \vee \neg q \vee r, 1), (\neg p \vee s, 1),\\ &(\neg p \vee \neg s \vee \neg r, 1), (\neg p \vee q \vee r \vee e, 1), (\neg p \vee s \vee e, 1), (\neg p \vee \neg s \vee \neg r \vee e, 1)\}.\end{aligned}$$

– Method 2: $\langle g^{Q_C, f^{\Sigma}}, \nabla^{wc} \rangle$ and $\oplus = Lukasiewicz\ t-norm$:
  Since $\mathcal{KP}$ is not consistent, we need to compute the distance from each PBB to others using $g^{Q_C, f^{\Sigma}}$. $Q_c(A, B) = 0.6$, $Q_C(A, C) = 0.4$, $Q_C(B, C) = 0.6$. So $f^{\Sigma}_{\mathcal{KP}}(A) = 1$, $f^{\Sigma}_{\mathcal{KP}}(B) = 1.2$, $f^{\Sigma}_{\mathcal{KP}}(C) = 1$. In the first round, $g^{Q_C, f^{\Sigma}}(\mathcal{KP}) = \{B\}$. So B is replaced by $\nabla^{wc}(B) = \{(q, 0.8), (e, 0.8)\}$. The obtained belief profile is still inconsistent, we must then go to the second round. Now $Q_C(A, B) = 0$, $Q_C(A, C) = 0.4$, $Q_C(B, C) = 0.6$. So $f^{\Sigma}_{\mathcal{KP}}(A) = 0.4$, $f^{\Sigma}_{\mathcal{KP}}(B) = 0.6$, $f^{\Sigma}_{\mathcal{KP}}(C) = 1$. So $g^{Q_C, f^{\Sigma}}(\mathcal{KP} = \{C\}$. C is then replaced by $\nabla^{wc}(C) = \{(\neg p, 0.8), (\neg q, 0.6)\}$. The obtained belief profile is inconsistent again, we must now go to the third round. $Q_C(A, B) = 0$, $Q_C(A, C) = 0.4$, $Q_C(B, C) = 0.6$. So $f^{\Sigma}_{\mathcal{KP}}(A) = 0.4$, $f^{\Sigma}_{\mathcal{KP}}(B) = 0.6$, $f^{\Sigma}_{\mathcal{KP}}(C) = 1$. So $g^{Q_C, f^{\Sigma}}(\mathcal{KP} = \{C\}$. C is then replaced by $\nabla^{wc}(C) = \{(\neg p, 0.8)\}$. Since the obtained belief profile is still inconsistent, we must go to the fourth round. Now $Q_C(A, B) = 0$, $Q_C(A, C) = 0.4$, $Q_C(B, C) = 0.6$. So $f^{\Sigma}_{\mathcal{KP}}(A) = 0.4$, $f^{\Sigma}_{\mathcal{KP}}(B) = 0$, $f^{\Sigma}_{\mathcal{KP}}(C) = 0.4$, and $g^{Q_C, f^{\Sigma}}(\mathcal{KP} = \{A, C\}$. A is then replaced by $\nabla^{wc}(A) = \{(q \to r, 1), (s, 0.8), (\neg s \to \neg r, 0.9)\}$ and C is replaced by $\nabla^{wc}(C) = \emptyset$. Finally C loses the game and gives up all the beliefs. The obtained belief profile is consistent, and the result of merging is

$$\begin{aligned}\mathcal{KP}_{\oplus} = \{&(q \to r, 1), (s, 0.8), (\neg s \to \neg r, 0.9), (q, 0.8), (e, 0.8), (q \vee s, 1), (\neg q \vee e \vee r, 1),\\ &(e \vee s, 1), (e \vee \neg s \vee r, 1)\}\end{aligned}$$

It is clear that the negotiation process in the second method is more complex than that of the first one. However, in the second merging method, C loses the game and gives up all its beliefs.

7 Conclusions

In this paper, we proposed a prioritized belief negotiation model which generalizes Konieczny's belief game model [12]. We then presented a two-step scenario

[2] To make the notation simpler, we will ignore the subscript of the weakening functions. Moreover, we don't use subscripts to denote the different weakening steps of the bases.

for merging PBBs based on the prioritized belief negotiation model. In the first step, original PBBs are weakened to make them consistent. Then in the second step, we combine the resulting PBBs using some combination rules in possibilistic logic [4]. Unlike the belief game model and Booth's belief negotiation model, our prioritized belief negotiation model takes into account the syntax of the PBBs and we have defined some particular negotiation functions and weakening functions by considering the priorities of formulae in each PBB.

8 Acknowledge

The authors are grateful to Richard Booth for his valuable comments on the draft paper.

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