Multiple Semi-Revision in Possibilistic Logic

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Abstract. Semi-revision is a model of belief change that differs from revision in that a new formula is not always accepted. Later, Fuhrmann defined multiple semi-revision by replacing a new formula with a set of formulae as the new information, which results in a merging operator called a partial meet merging operator. The problem for the partial meet merging operator is that it needs additional information to define a *selection function* which selects a subset from a set of maximal consistent subbases of an inconsistent knowledge base. In this paper, we will extend multiple semi-revision in the framework of possibilistic logic. The advantage of possibilistic logic is that it provides an ordering relation on formulae in knowledge bases, which makes it easy to define a *selection function* practically.

1 Introduction

The problem of belief revision has been widely discussed in the past twenty years [1, 8, 11, 12, 19, 24]. In belief revision theory, new information (a new formula) must be adopted and some existing information will be dropped to accommodate it. However, many researchers argued that new information should not always have the priority over the existing information and some non-prioritized belief revision methods have been proposed in which new informatin is not necessarily accepted [11, 17, 18]. For example, the semi-revision introduced by Hansson [17] differs from belief revision in two aspects: first, original information is represented as a belief base rather than a belief set, and second, new information is not always accepted. The semi-revision can be related to belief merging which deals with the problem of deriving a coherent belief base from a set of inconsistent belief bases [2–5, 11, 13–15, 19, 23]. Fuhrmann in [11] considered a multiple semi-revision by replacing the new formula with a set of formulae as new information, which results in a merging operator which he called a *partial meet merging operator*. Both the semi-revision and the partial meet merge methods consist of two steps. The first step is to conjoin original information and new information and the second step is to restore consistency using a contraction function defined in [1, 16].

Two problems exist in semi-revision and partial meet merge. First, it is not advisable to conjoin an original knowledge base with a new formula (or a set of formulae) because some information may be lost. Let us look at an example. Let $K_1 = \{\phi, \psi\}$ be the original knowledge base and $K_2 = \{\phi, \psi\}$ be the new information. Conjoining K_1 and K_2 results in $K_3 = \{\phi, \psi\}$, which is consistent. It is the result of partial meet merge of K_1 and K_2. If new information $K_4 = \{\neg\phi\}$ is obtained, conjoining K_3 and K_4 results in a knowledge base $K = \{\phi, \neg\phi, \psi\}$. Since ϕ and $\neg\phi$ are equally reliable, it is not possible to decide which formula should be dropped, both ϕ and $\neg\phi$ have to be deleted. However, since both K_1 and K_2 support ϕ and only K_4 supports $\neg\phi$, by majority principle, ϕ should be kept and $\neg\phi$ should be deleted.

The second problem is that we need a method to define a contraction function in a practical way. In belief revision [1, 12], this problem is solved by considering a notion of epistemic entrenchment. An epistemic entrenchment is an ordering that envisages the logical dependencies of the formulae in the belief set. It is the epistemic entrenchment of a formula in a belief set that determines the formula's fate when the belief set is contracted.

In this paper, we will resolve above problems by considering the multiple semi-revision in possibilistic logic. In [9], a corresponding relationship between epistemic entrenchment and possibilistic logic has been established. It has been shown that the only numerical counterparts of epistemic entrenchment relations are necessity measures. Possibilistic logic is an extension of classical logic. Each formula in possibilistic logic is attached with a weight denoting its necessity degree. Possibilistic logic has been shown to be a good framework for belief revision and belief merging [3–5].

Multiple semi-revision in possibilistic logic is carried out in two steps: a combination step and an inconsistency handling step. In the combination step, each belief base is split into two subbases: one consists of *conflict* formulae and the other consists of *free* formulae in the union of all the belief bases. The weights of formulae in the subbases with *free* formulae are either increased or unchanged and the weights of formulae in the subbases with *conflict* formulae are decreased. That is, we have a *reinforcement* effect on the *free* formulae and a *counteract* effect on the *conflict* formulae. This method is more reasonable than the conjoining method because it does not ignore any information in both sources. Then in the inconsistency handling step, we will restore consistency of the resulting belief base if it is inconsistent by dropping some conflict formulae according to their priorities.

This paper is organized as follows. Section 2 gives a brief review of possibilistic logic. We then introduce Hansson's semi-revision and Fuhrmann's partial meet merging in Section 3. In Section 4 we will define the stratified semi-revision. We compare the stratified semi-revision and some other merging methods in possibilistic logic in Section 5. Finally, we conclude this paper in Section 6.

2 Possibilistic Logic

In this paper, we only consider a finite propositional language denoted by $\mathcal{L}$. The classical consequence relation is denoted as $\vdash$. ϕ, ψ, γ,... represent classical formulas. $\top$ and $\perp$ represent constant truth and constant false respectively. A (classical) *knowledge base* K is a finite set of propositional formulas. Knowledge bases are denoted by capital letters A, B, C, K...

Possibilistic logic [10] is an extension of classical logic. It is a weighted logic where each classical formula is associated with a level of priority. A possibilistic knowledge base is the set of possibilistic formulae of the form $\mathcal{B} = \{(\phi_i,\ a_i) : i = 1, ..., n\}$. The *possibilistic formula* $(\phi_i,\ a_i)$ means that the necessity degree of ϕ_i is at least equal to a_i. Let $\mathcal{KB}$ denote the set of all the possibilistic knowledge bases. In this paper, we only consider possibilistic knowledge bases where every formula ϕ is a classical propositional formula. The classical base associated with $\mathcal{B}$ is denoted as $\mathcal{B}^*$, namely $\mathcal{B}^* = \{\phi_i|(\phi_i,\ a_i) \in \mathcal{B}\}$. A *possibilistic base* $\mathcal{B}$ is consistent if and only if its classical base $\mathcal{B}^*$ is consistent. The formulas in $\mathcal{B}$ can be rearranged according to their weights in the descending order, such that $a_0 = 1 \geq a_1 \geq ... \geq a_n > 0$. Suppose there are m distinct a_i values (weights) $a_{i_1}, ..., a_{i_m}$, where $a_{i_j} > a_{i_{j+1}}$. Then $\mathcal{B}$ can be equivalently expressed as a layered belief base $\Sigma_{\mathcal{B}} = S_1 \cup ... \cup S_m$, where $S_k = \{\phi : (\phi, a_{i_k}) \in \mathcal{B}\}$. $\Sigma_{\mathcal{B}}$ is called the stratification of $\mathcal{B}$.

In possibilistic logic, a *possibility distribution*, denoted by π, is a mapping from a set of possible worlds $\mathcal{W}$ to the interval [0,1]. $\pi(\omega)$ represents the possibility degree of the interpretation ω with the available beliefs. From a *possibility distribution* π, two measures defined on a set of propositional or first order formulas can be determined. One is the possibility degree of formula ϕ, denoted as $\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$. The other is the necessity degree of formula ϕ, and is defined as $N(\phi) = 1 - \Pi(\neg\phi)$.

Definition 1. *[10] Let $\mathcal{B}$ be a PKB, and $\alpha \in [0, 1]$. The* α-cut *of $\mathcal{B}$ is $\mathcal{B}_{\geq\alpha} = \{\phi \in \mathcal{B}^* | (\phi, a) \in \mathcal{B}$ and $a \geq \alpha\}$.*

Definition 2. *[4] A subbase $\mathcal{A}$ of $\mathcal{B}$ is said to be minimally inconsistent if and only if it satisfies the following two requirements:*

- *$(\mathcal{A})^* \models \perp$, where $(\mathcal{A})^*$ is the classical base of $\mathcal{A}$, and*
- *$\forall \phi \in (\mathcal{A})^*$, $(\mathcal{A})^* - \{\phi\} \not\models \perp$.*

Definition 3. *[4] A possibilistic formula (ϕ, a) is said to be $free$ in $\mathcal{B}$ if it does not belong to any minimally inconsistent subbase of $\mathcal{B}$ and (ϕ, a) is said to be $conflict$ in $\mathcal{B}$ otherwise. $Conflict(\mathcal{B})$ to denote the set of formulae in $\mathcal{B}$ which are in conflict.*

The *inconsistency degree* of $\mathcal{B}$, which defines the level of inconsistency of $\mathcal{B}$, is defined as [10]:

$$Inc(\mathcal{B}) = max\{\alpha_i | \mathcal{B}_{\geq\alpha_i} \textit{ is inconsistent}\}.$$

Suppose $\Sigma_{\mathcal{B}}$ is the stratification of $\mathcal{B}$, then the degree of inconsistency of $\Sigma_{\mathcal{B}}$ is defined as the degree of inconsistency of $\mathcal{B}$.

Definition 4. *[10] Let $\mathcal{B}$ be a possibilistic base. Let $(\phi,\ \alpha)$ be a piece of information with $\alpha > Inc(\mathcal{B})$. $(\phi,\ \alpha)$ is said to be a consequence of $\mathcal{B}$, denoted by $\mathcal{B} \vdash_\pi (\phi,\ \alpha)$, iff $\mathcal{B}_{\geq\alpha} \vdash \phi$.*

3 Semi-Revision

The main difference between semi-revision [17] and traditional belief revision [1, 12] is that a new formula is not necessily accepted. The basic idea of semi-revision is to conjoin the original belief base and the new formula and then drop some formulae in the resulting base to restore consistency.

Definition 5. *[1] Let A be a set of formulae and ϕ a formula. The set $A\bot\phi$ [1](“A less ϕ”) is the set of sets such that $B\in A\bot\phi$ if and only if:*

(1) $B\subseteq A$
(2) $B\nvdash\phi$
(3) $\forall B'\subset A$*, if* $B\subset B'$*, then* $B'\vdash\phi$

Definition 6. *[1] A* selection *function for a set A of formulae is a function γ such that for every formula ϕ:*

(1) If $A\bot\phi$ is non-empty, then $\gamma(A\bot\phi)$ is a non-empty subset of $A\bot\phi$, and
(2) If $A\bot\phi$ is empty, then $\gamma(A\bot\phi) = \{A\}$.

Definition 7. *[1] Let A be a set of formulae and γ a selection function for A. The* partial meet contraction *on A that is generated by γ is the operation $\sim\gamma$ such that for every formula ϕ:*

$$A\sim\gamma\phi = \cap\gamma(A\bot\phi)$$

Partial meet semi-revision [17] is based on the partial meet contraction. It first adds the belief ϕ to the base, and then the resulting base is contracted by the constant false $\bot$.

Definition 8. *The* partial meet semi-revision *of B based on a selection function γ is the operator $?_\gamma$ such that for every fomula ϕ:*

$$B?_\gamma\phi = (B\cup\{\phi\})\sim\gamma\bot = \cap\gamma((B\cup\{\phi\})\bot\{\bot\})$$

In [11], Fuhrmann generalized the semi-revision by replacing the input as a set of formulae, which results in a merging operator.

Definition 9. *Let A and B be two belief bases. The partial meet merge of A and B is defined as:*

$$A\circ B = (A\cup B)\sim\gamma\bot$$

Fuhrmann also gave the axiomatic characterization of the partial meet merge [11].

Theorem 1. *$\circ$ is an operator of partial meet merge if and only if it satisfies:*

(M1) *$A\circ B$ is consistent (*strong consistency*)*
(M2) *$A\circ B\subseteq A\cup B$ (*inclusion*)*
(M3) *If $\phi\in(A\cup B)\setminus(A\circ B)$, Then $\exists D : A\circ B\subseteq D\subseteq A\cup B$ and $D\nvdash\bot$ but $D\cup\{\phi\}\vdash\bot$ (*relevance*)*
(M4) *If $A\cup B = A'\cup B'$, then $A\circ B = A'\circ B'$ (*congruence*)*

[1] We use $\bot$ to denote both the constant false and the operation to obtain the set of maximal subbases of A which do not imply ϕ as in belief revision literature. Hopefully it will not make confusion.

4 Multiple Semi-Revision: A Possibilistic Approach

Priority plays a very important role in belief revision [12, 19]. Possibilistic logic is a good framework to represent priority [4]. In this section, we extend multiple semi-revision in the framework of possibilistic logic. Multiple semi-revision consists of two steps: one is the combination step and the other is the inconsistency handling step. In the combination step, the original knowledge bases are combined that may produce a possibly inconsistent knowledge base. In the inconsistency handling step, some conflicting formulae are dropped to restore consistency.

4.1 Combination step

In semi-revision and partial meet merge, the intermediate combination step is to conjoin original information and new information. Recall the example given in the Section 1, a disadvantage of conjoining the knowledge bases is that some important information may be lost.

It is also not always advisable to conjoin two possibilistic knowledge bases in the intermediate combination step in multiple semi-revision in possibilistic logic. Let us look at an example to illustrate the reason for it.

Example 1. Let $\mathcal{B}_1 = \{(\neg\phi, 0.7), (\gamma, 0.8)\}$ and $\mathcal{B}_2 = \{(\phi, 0.6), (\gamma, 0.8)\}$ be two possibilistic belief bases. By conjoining them we obtain a knowledge base $\mathcal{B} = \{(\neg\phi, 0.7), (\phi, 0.6), (\gamma, 0.8)\}$. Since the weight of $\neg\phi$ is greater than that of ϕ, it is reasonable to delete ϕ, so the result of merging is $\mathcal{B}_3 = \{(\neg\phi, 0.7), (\gamma, 0.8)\}$. Suppose later we receive another source represented as $\mathcal{B}_4 = \{(\phi, 0.7), (\neg\gamma, 0.8)\}$. By conjoining $\mathcal{B}_3$ and $\mathcal{B}_4$ we get $\mathcal{B}' = \{(\neg\phi, 0.7), (\phi, 0.7), (\gamma, 0.8), (\neg\gamma, 0.8)\}$. Since ϕ, $\neg\phi$ have the same weights and γ, $\neg\gamma$ have the same weights, we have to drop all the formulae in $\mathcal{B}'$. So the final result is a knowledge base with no information. This is not reasonable! For ϕ, there are two sources $\mathcal{B}_1$ and $\mathcal{B}_2$ supporting it with weights 0.7 and 0.6 respectively. Whilst there is only one source supporting $\neg\phi$ with weight 0.7. So we may prefer to retain ϕ and drop $\neg\phi$. For the same reason, it is more reasonable to retain γ and drop $\neg\gamma$. The problem for the example above is that when we combine $\mathcal{B}_1$ and $\mathcal{B}_2$ by conjoining them, after restoring consistency, information provided by $\mathcal{B}_2$ is ignored.

Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be two possibilistic knowledge bases from two different sources. For those formulas that are involved in the conflict in $\mathcal{B}_1 \cup \mathcal{B}_2$, their necessity degrees should decrease after combination because they will counteract with each other. In contrast, the necessity degree should increase for those formulas that are supported by both sources.

Definition 10. *[6] An operator $\oplus_{SC}$ is said to be strongly conjunctive on [0,1] if for all $(a_1, ..., a_n)$*

$$\oplus_{SC}(a_1, ..., a_n) \geq max(a_1, ..., a_n).$$

A strongly conjunctive operator is used to increase the weight of a formula after combination. Many operators belong to this class, such as the probabilistic sum $\oplus(a,b) = min(a+b-ab,1)$ and bounded sum $\oplus(a,b) = min(a+b,1)$.

Definition 11. *[21] An operator $\oplus_{UA}$ is said to be an up-averaging operator if for all $(a_1,...,a_n)$*

$$\oplus_{UA}(a_1,...,a_n)\leq max(a_1,...,a_n).$$

This operator reflects that a combination result cannot be greater than the greatest of all. An example of up-average operator is the standard average operator $\oplus(a,b) = (a+b)/2$. Another up-average operator, called *max-product* operator, is defined as follows:

$$\oplus_{max,pro}(a,b) = \begin{cases} max(a,b) & \text{if } a,b\neq 0, \\ max(a^2,b^2) & \text{otherwise.} \end{cases}$$

This operator reflects that if a formula is supported by two sources with weights greater than 0, then we keep the maximum weight as the result of combination of two weights a and b, otherwise the weight of the formula will be decreased after combination.

Now we give a combination method based on the operators defined above.

Given two knowledge bases $\mathcal{B}_1$ and $\mathcal{B}_2$, we use two operators, one is a strongly conjunctive operator and the other is an up-averaging operator. For those formulas that are not in conflict in $\mathcal{B}_1\cup\mathcal{B}_2$, we choose the strongly conjunctive operator to combine them. But for those formulas that are in conflict, we use the up-averaging operator to combine them. We always assume that if a formula ϕ does not appear in a possibilistic knowledge base $\mathcal{B}$, then $(\phi,0)$ has been added to $\mathcal{B}$ implicitly if necessary. Moreover, we assume that each formula in a possibilistic knowledge base appears only once with a unique weight.

Definition 12. *Let $\mathcal{B}_1 = \{(\phi_i,a_i) : i = 1,...,n\}$ and $\mathcal{B}_2 = \{(\psi_j,b_j) : j = 1,...,m\}$ be two self-consistent possibilistic knowledge bases. Let $\oplus_{SC}$ and $\oplus_{UA}$ be a strong conjunctive operator and an up-averaging operator respectively. The combination of $\mathcal{B}_1$ and $\mathcal{B}_2$ is defined as $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2) = \mathcal{C}\cup\mathcal{D}$, where*

$$\mathcal{C} = \{(\phi,\oplus_{UA}(a,b))|\phi\in(Conflict(\mathcal{B}_1\cup\mathcal{B}_2))^*, (\phi,a)\in\mathcal{B}_1\ (\phi,b)\in\mathcal{B}_2\},$$

$$\mathcal{D} = \{(\phi,\oplus_{SC}(a,b))|\phi\notin(Conflict(\mathcal{B}_1\cup\mathcal{B}_2))^*, (\phi,a)\in\mathcal{B}_1 \text{ and } (\phi,b)\in\mathcal{B}_2\}$$

Example 2. (Continue Example 1) Since γ is supported by both sources, its certainty degree should increase, i.e., there is a reinforcement between $\mathcal{B}_1$ and $\mathcal{B}_2$ for γ. For formulas ϕ and $\neg\phi$, they are involved in the inconsistency of $\mathcal{B}_1\cup\mathcal{B}_2$, so their necessity degrees should decrease. Let $\oplus_{SC}$ be the probabilistic sum and $\oplus_{UA}$ be the *max-product* operator. By Definition 12, the combination of $\mathcal{B}_1$ and $\mathcal{B}_2$ is $\mathcal{B} = \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2) = \{(\neg\phi,0.49),(\phi,0.36),(\gamma,0.96)\}$.

4.2 Inconsistency handling step

The knowledge base obtained by the combination step is inconsistent if the original knowledge bases are in conflict. As in semi-revision and partial meet merge, we will drop some formulae in the knowledge base to restore consistency. Since a possibilistic knowledge base provides explicit priorities between formulae, we can drop those formulae in conflict according to their weights or priorities.

As most inconsistency handling methods in possibilistic logic [4], first we need to stratify the possibilistic knowledge bases. A very common approach to handling inconsistency in a stratified knowledge base is to keep as much information in a higher layer as possible.

Definition 13. *[4] Let $\Sigma = S_1 \cup ... \cup S_n$ be a layered belief base. A subbase $\Sigma' = A_1 \cup ... \cup A_n$ of Σ is a strongly maximal consistent subbase (SMC-subbase for short) iff for all k $(1 \le k \le n)$ $A_1 \cup ... \cup A_k$ is a maximal consistent subset of $S_1 \cup ... \cup S_k$. The set of all SMC-subbases of Σ is denoted by $SMC(\Sigma)$.*

The SMC-subbase was also defined in [7], with the name *"preferred subbases"*. It can be constructed by starting with a maximal consistent subset of S_1, then adding to the maximal consistent subset as many formulas of S_2 as possible (while preserving consistency), and so on. So a SMC-subbase Σ' of a stratified belief base Σ must be a maximal subbase of it, i.e., $\Sigma' \in \Sigma \bot \{\bot\}$. The following proposition suggests that SMC-subbases are acceptable in the sense of the best out selection.

Proposition 1. *[4] Let $\Sigma_{\mathcal{B}}$ be the stratification of a possibilistic knowledge base $\mathcal{B}$. A SMC-subbase of $\Sigma_{\mathcal{B}} = S_1 \cup ... \cup S_n$ is $\Sigma' = A_1 \cup ... \cup A_n$ such that the degree of inconsistency of $\Sigma' \cup \{\phi\}_i$ is a_i, $\forall \phi \in S_i - A_i$, where $\Sigma' \cup \{\phi\}_i$ is the new stratified knowledge base obtained by adding ϕ to the layer S_i in Σ'.*

Now suppose we have two possibilistic knowledge bases $\mathcal{B}_1$ and $\mathcal{B}_2$, where $\mathcal{B}_1$ is the original knowledge base the $\mathcal{B}_2$ is a new knowledge base. Then the multiple semi-revision is processed as follows. First we combine $\mathcal{B}_1$ and $\mathcal{B}_2$ as $\Delta_{\oplus_{SC}, \oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2)$. Let Σ be the stratification of $\Delta_{\oplus_{SC}, \oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2)$, then in the second step, we delete those elements of $\Delta_{\oplus_{SC}, \oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2)$ that do not belong to any of the elements of $SMC(\Sigma)$.

Definition 14. *Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be two possibilistic knowledge bases. $\Delta_{\oplus_{SC}, \oplus_{UA}}$ $(\mathcal{B}_1, \mathcal{B}_2)$ is the possibilistic knowledge base obtained by the combination step. Suppose Σ is the stratification of $\Delta_{\oplus_{SC}, \oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2)$. Let $\Sigma' = A_1 \cup ... \cup A_n = \cap \{\Sigma_i \subseteq \Sigma : \Sigma_i \in SMC(\Sigma)\}$. Then the SMC-subbases based merging is defined as $\mathcal{B}_1 \circ^{SMC}_{\oplus_{SC}, \oplus_{UA}} \mathcal{B}_2 = \{(\phi, a_i) \in \Delta_{\oplus_{SC}, \oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2) : \phi \in A_i\}$.*

Example 3. (Continue Example 2) In Example 2, the combination of $\mathcal{B}_1$ and $\mathcal{B}_2$ is $\mathcal{B} = \Delta_{\oplus_{SC}, \oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2) = \{(\neg\phi, 0.49), (\phi, 0.36), (\gamma, 0.96)\}$. The stratification of $\mathcal{B}$ is $\Sigma_{\mathcal{B}} = \{\{\gamma\}, \{\neg\phi\}, \{\phi\}\}$, and the only SMC-subbase of $\Sigma_{\mathcal{B}}$ is $\{\{\gamma\}, \{\neg\phi\}\}$. So the result of merging of $\mathcal{B}_1$ and $\mathcal{B}_2$ is $\mathcal{B}_3 = \{(\neg\phi, 0.49), (\gamma, 0.96)\}$. Now suppose another source $\mathcal{B}_4 = \{(\phi, 0.7), (\neg\gamma, 0.8)\}$ is received. By combining $\mathcal{B}_3$ and $\mathcal{B}_4$ we get $\mathcal{B}' = \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_3, \mathcal{B}_4) = \{(\neg\phi, 0.24), (\phi, 0.49), (\gamma, 0.92), (\neg\gamma, 0.64)\}$. The stratification of $\mathcal{B}'$ is $\Sigma_{\mathcal{B}'} = \{\{\gamma\}, \{\neg\gamma\}, \{\phi\}, \{\neg\phi\}\}$. The only SMC-subbase of $\Sigma_{\mathcal{B}'}$ is $\{\{\gamma\}, \{\phi\}\}$. So the final result of merging is $\mathcal{B}_5 = \{(\phi, 49), (\gamma, 0.92)\}$. So both ϕ and γ can be inferred from $\mathcal{B}_5$, which is consistent with our analysis in Example 1.

Example 4. Let $\mathcal{B}_1 = \{(\phi, 0.8), (\neg\phi \vee \psi, 0.7), (\gamma, 0.6), (\psi \vee \varphi, 0.5)\}$ and $\mathcal{B}_2 = \{(\neg\phi, 0.8), (\neg\psi, 0.7), (\gamma, 0.7)\}$. Let $\oplus_{SC}$ be the probabilistic sum and $\oplus_{UA}$ be the *max-product* operator. The knowledge base obtained by the combination step is $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2) = \{(\gamma, 0.88), (\phi, 0.64), (\neg\phi, 0.64), (\neg\phi\vee\psi, 0.49), (\neg\psi, 0.49), (\varphi\vee\psi, 0.5)\}$. The stratification of $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2)$ is $\Sigma = \{\{\gamma\}, \{\phi, \neg\phi\}, \{\varphi \vee \psi\}, \{\neg\phi\vee\psi, \neg\psi\}\}$. There are three SMC-subbases in Σ: $\{\{\gamma\}, \{\phi\}, \{\varphi\vee\psi\}, \{\neg\phi\vee\psi\}\}$, $\{\{\gamma\}, \{\phi\}, \{\varphi \vee \psi\}, \{\neg\psi\}\}$, $\{\{\gamma\}, \{\neg\phi\}, \{\varphi \vee \psi\}, \{\neg\phi \vee \psi, \neg\psi\}\}$. The intersection of the SMC-subbases is $\{\{\gamma, \}, \{\varphi\vee\psi\}\}$. So the result of SMC-subbases based merge of $\mathcal{B}_1$ and $\mathcal{B}_2$ is $\mathcal{B} = \{(\gamma, 0.88), (\varphi \vee \psi, 0.5)\}$.

The SMC-subbases based merge discards too much information. In Example 4, all the formulae involved in conflict are dropped after merging. As in the semi-revision and partial meet merge, we can select a subset of SMC-subbases. This can be done by defining a selection function as follows.

Definition 15. *A* selection function *for a layered belief base Σ is a function γ such that:*

(1) If $SMC(\Sigma)$ is non-empty, then $\emptyset\subset\gamma(SMC(\Sigma))\subseteq SMC(\Sigma)$, and
(2) If $SMC(\Sigma)$ is empty, then $\gamma(SMC(\Sigma)) = \{\Sigma\}$.

The merging operator based on a selection function is defined as follows.

Definition 16. *Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be two possibilistic knowledge bases. $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2)$ is the possibilistic knowledge base obtained by the combination step. Suppose Σ is the stratification of $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2)$. Let γ be a selection function for Σ. Let $\Sigma' = A_1\cup...\cup A_n = \cap\{\Sigma_i\subseteq\Sigma : \Sigma_i\in\gamma(SMC(\Sigma))\}$. The partial SMC-subbases based merging is defined as $\mathcal{B}_1 \circ^{PSMC}_{\oplus_{SA},\oplus_{UA}} \mathcal{B}_2 = \{(\phi, a_i)\in \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1, \mathcal{B}_2) : \phi\in A_i\}$.*

A particular selection function can be defined by selecting the lexicographically maximal consistent subbases [4].

Definition 17. *[4] Let Σ be a stratified knowledge base. Suppose $SMC(\Sigma)$ is the set of SMC-subbases of Σ, then any $\Sigma' = A_1\cup...\cup A_n\in SMC(\Sigma)$ is said to be a lexicographically maximal consistent (LMC) subset of Σ if and only if $\forall\Sigma'' = B_1\cup...\cup B_n\in SMC(\Sigma)$,*

$$\nexists i, such\ that |B_i| > |A_i|\ and\ \forall j < i, |B_j| = |A_j|$$

The set of all lexicographically maximal consistent subsets of Σ is denoted as $Lex(\Sigma)$.

Definition 18. *Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be two possibilistic knowledge bases. $\Delta_{\oplus_{SC},\oplus_{UA}}$ $(\mathcal{B}_1,\mathcal{B}_2)$ is the possibilistic knowledge base obtained by the combination step. Suppose Σ is the stratification of $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$. Let $\Sigma' = A_1 \cup ... \cup A_n = \cap\{\Sigma_i \subseteq \Sigma : \Sigma_i \in Lex(\Sigma)\}$. Then the Lex-subbases based merging is defined as $\mathcal{B}_1 \circ^{Lex}_{\oplus_{SA},\oplus_{UA}} \mathcal{B}_2 = \{(\phi, a_i) \in \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2) : \phi \in A_i\}$.*

Example 5. (Continue Example 4) The lexicographically maximal consistent subbase of Σ is $\{\{\gamma\},\{\neg\phi\},\{\varphi\vee\psi\},\{\neg\phi\vee\psi,\neg\psi\}\}$. So the result of *Lex*-subbases based merge is $\mathcal{B} = \{(\gamma, 0.88), (\neg\phi, 0.64), (\neg\phi \vee \psi, 0.49), (\neg\psi, 0.49), (\varphi \vee \psi, 0.5)\}$, which is equivalent to $\mathcal{B}' = \{(\gamma, 0.88), (\neg\phi, 0.64), (\neg\psi, 0.49), (\varphi \vee \psi, 0.5)\}$.

In Example 5, $\mathcal{B}'$ contains two more formulae $(\neg\phi, 0.64)$ and $(\neg\psi, 0.49)$ than $\mathcal{B}$ in Example 4. Although ϕ and $\neg\phi$ have the same priority, both formulae $\neg\phi \vee \psi$ and $\neg\psi$ from the lower level give support to $\neg\phi$. So we still accept $\neg\phi$ and drop ϕ.

5 Postulates for Partial *SMC*-subbases based Merge

In this section, we will propose the postulates to characterize the partial SMC-subbases based merge by adapting the postulates for partial meet merge in Theorem 1.

First, by Definition 16, the condition *strong consistency* still holds for the partial SMC-subbases based merging operator. However, other postulates should be changed because we do not conjoin the knowledge bases in the combination step. There are two main differences between the partial SMC-subbases based merging operator and the partial meet merging operator. Fist, given two possibilistic knowledge bases $\mathcal{B}_1$ and $\mathcal{B}_2$, instead of conjoining them, we take $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$ as the result of combination step. Second, the partial SMC-subbases based merging operator is based on a selection function which selects a subset of the set of SMC-subbase of Σ, the stratification of $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$. So we have the following postulates for the partial SMC-subbases based merging operator.

Theorem 2. *Let $\oplus_{SA}$ and $\oplus_{UA}$ be a strongly conjunctive operator and an upper-averaging operator respectively. An operator $\circ$: $\mathcal{KB}\times\mathcal{KB}\rightarrow\mathcal{KB}$ is a partial SMC-subbases based merging operator with regard to $\oplus_{SA}$ and $\oplus_{UA}$ iff for every two possibilistic knowledge bases $\mathcal{B}_1$ and $\mathcal{B}_2$, it satisfies the following conditions:*

1. *$(\mathcal{B}_1 \circ \mathcal{B}_2)^* \nvdash \bot$ (consistency)*
2. *$\mathcal{B}_1 \circ \mathcal{B}_2 \subseteq \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$ (inclusion)*
3. *If $(\phi, a) \in \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$ and $(\phi, a) \notin \mathcal{B}_1 \circ \mathcal{B}_2$, then $\exists\mathcal{E}$ such that $\mathcal{B}_1 \circ B_2 \subseteq \mathcal{E} \subseteq \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$, and $\mathcal{E}^* \nvdash \bot$ and $Inc(\mathcal{E} \cup \{(\phi, a)\}) = a$.*
4. *If $\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2) = \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}'_1,\mathcal{B}'_2)$, then $\mathcal{B}_1 \circ \mathcal{B}_2 = \mathcal{B}'_1 \circ \mathcal{B}'_2$.*

Proof. We only prove the "only if" part, the proof of "if" part is similar to that of Theorem 1 [11].

($\Longrightarrow$) Conditions 1, 2 and 4 clearly hold. To prove Condition 3, let us assume $(\phi,a)\in\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$ *and* $(\phi,a)\notin\mathcal{B}_1\circ\mathcal{B}_2$. *Let* $\Sigma = S_1\cup...\cup S_n = \Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$ *and* $\phi\in S_k$. *By Definition 16, there is some* $\Sigma'\in\gamma(SMC(\Sigma))$ *such that* $\Sigma' = A_1\cup...\cup A_n$ *and* $\phi\notin A_k$. *Let* $\mathcal{E} = \{(\phi,a_i)\in\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2) : \phi\in A_i, 1\leq i\leq n\}$. *It is clear that* $\mathcal{B}_1\circ B_2\subseteq\mathcal{E}\subseteq\Delta_{\oplus_{SC},\oplus_{UA}}(\mathcal{B}_1,\mathcal{B}_2)$, *and* $\mathcal{E}^*\not\vdash\perp$. *Since* $\phi\in S_k-A_k$, *by Proposition 1,* $Inc(\mathcal{E}\cup\{(\phi,a)\}) = a$.

By Condition 3 above, a formula which is deleted after merging must be in conflict in $\mathcal{B}_1\cup\mathcal{B}_2$. Since in the combination step, the weights of free formulae will increase or keep intact, we have the following corollary.

Corollary 1. *Let* $\oplus_{SA}$ *and* $\oplus_{UA}$ *be a strongly conjunctive operator and an upper-averaging operator respectively. Let* $\mathcal{B}_1$ *and* $\mathcal{B}_2$ *be two possibilistic knowledge bases, with* $\mathcal{B}_1\circ\mathcal{B}_2$ *as the result of merging by a partial* SMC*-subbase based merging operator with regard to* $\oplus_{SA}$ *and* $\oplus_{UA}$. *If* (ϕ,a) *is a free formula in* $\mathcal{B}_1\cup\mathcal{B}_2$, *then* $(\phi,b)\in\mathcal{B}_1\circ\mathcal{B}_2$ *and* $b\geq a$.

The following Corollary tells us that our partial SMC-subbases based merging operator is a generalization of Fuhrmann's partial meet merging operator.

Corollary 2. *Let* $\mathcal{B}_1$ *and* $\mathcal{B}_2$ *be two classical knowledge bases. Let* $\oplus_{SC}(a,b) = \oplus_{UA}(a,b) = max(a,b)$. *Then the partial* SMC*-subbases based merging operator* $\circ_{\oplus_{SA},\oplus_{UA}}$ *and partial meet merging operator* $\circ$ *have the same result, i.e.* $\mathcal{B}_1\circ_{\oplus_{SA},\oplus_{UA}}\mathcal{B}_2 = \mathcal{B}_1\circ\mathcal{B}_2$.

6 Related Work

Many merging operators have been proposed in the framework of possibilistic logic [3–6, 20, 21].

The merging operators in [3, 5] are defined semantically and syntactically, i.e. the fusion of two possibilistic knowledge bases are defined semantically by combining their possibility distributions using an operator which is weakly constrained (the result is a new possibility distribution) and then a possibilistic knowledge base is recovered from the new possibility distribution. A problem is, if the result of merging is required to be consistent, *disjunctive* operators are usually chosen, which was criticized to be too cautious.

In [20], we proposed a split-combination method for merging possibilistic knowledge bases which combines formulae in conflict using a *disconjunctive* operator and formulae that are free using a *conjunctive* operator. We showed that this method improves the *disjunctive*-operator based methods because more information was kept after merging. A common point between the partial SMC-subbases based method and the split-combination method is that they both differentiate conflict formulae from free formulae and combines them using different operators. The difference among them is that the partial SMC-subbases based method resolves inconsistency by deleting some formulae that are in conflict whilst the split-combination method does this by weakening conflict information instead of deleting some of them.

Some inconsistency-tolerant consequence relations were proposed to deal with inconsistency in [4], merging uncertain sources of information is done in two steps: the first step is simply to conjoin the original knowledge bases, and then in the second step, an inconsistency-tolerant consequence will be applied to handle inconsistency. This method does not require to restore consistency after combination. Moreover, it conjoins the original knowledge bases, which is different from our first step of merging.

7 Conclusion

In this paper, we extend Fuhrmann's partial meet merge in possibilistic logic. The merge is processed in two steps: a combination step and an inconsistency handling step. In the combination step, we combine *free* formulae and *conflict* formulae using different operators. The result of combination in the first step may be an inconsistent knowledge base. Then in the inconsistency handling step, we delete those formulae that are in conflict and do not belong to some *strongly maximal consistent* subbase.

We only defined the merging operator for two knowledge bases. A future work is to extend it to merge more than two knowledge bases. A problem with it is that the order of merging will influence the final result. This problem exists in most merging methods. We will deal with this problem by introducing some criterion to decide which two knowledge bases should be merged first. For example, we can choose two knowledge bases which are "closest" to each other to merge each time.

Another important issue is how to choose the appropriate operators in the combination step. We have discussed some criteria to choose operators in [22]. More work will be done on this problem in the future.

References

1. Alchourrón, C.E., Gärdenfors, P., Markinson, D.: On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic*, vol. 50, pp. 510-530, 1985.
2. Baral, C., Kraus., Minker J., and Subrahmanian, V.S.: Combining knowledge bases consisting in first order theories. *Computational Intelligence*, vol. 8, pp. 45-71, 1992.
3. Benferhat, S., Dubois, D., Prade, H., and Williams, M.A.: A practical approach to fusing prioritized knowledge bases. In *Proc. of EPIA'99*, pp. 223-236, 1999.
4. Benferhat, S., Dubois, D., Prade, H.: Some syntactic approaches to the handling of inconsistent knowledge bases: A comparative study. Part 2: The prioritized case. In *Logic at work : essays dedicated to the memory of Helena Rasiowa* / Ewa Orow. - New York : Physica-Verlag, pp. 473-511, 1998.
5. Benferhat, S., Dubois, D., Kaci, S., and Prade, H.: Possibilistic merging and distance-based fusion of propositional information. *Annals of Mathematics and Artificial Intelligence*, vol. 34, pp. 217-252 , 2002.

6. Benferhat S.; Kaci S.: Fusion of possibilitic knowledge bases from a postulate point of view. *International Journal of Approximate Reasoning*, vol. 33(3), pp. 255-285, 2003.
7. Brewka G.: *Preferred subtheories: an extended logical framework for default reasoning*. In *Proc. of IJCAI'89*, pp. 1043-1048, 2003.
8. Chopra, S., Ghose A., Meyer T.: Non-prioritized ranked belief change. *Journal of Philosophical Logic*, vol. 32(4), pp. 417-443, 2003.
9. Dubois, D., Prade, H.: Epistemic entrenchment and possibilistic logic. *Artificial Intelligence*, vol. 50, pp. 223-239, 1991.
10. Dubois, D., Lang, J., and Prade, H.: Possibilistic logic. In *Handbook of logic in Aritificial Intelligence and Logic Programming*, Volume 3. Oxford University Press, pp. 439-513, 1994.
11. Fuhrmann, A.: *An Essay on Contraction*. Stanford University: CSLI Publications & FoLLI. 1996.
12. Gärdenfors P.: *Knowledge in Flux-Modeling the Dynamic of Epistemic States*. Mass.: MIT Press. 1988.
13. Konieczny, S., Pino Pérez, R.: On the logic of merging. In *Proc. of KR'98*, pp. 488-498, 1998.
14. Liberatore, P., and Schaerf, M.: Arbitration (or How to Merge Knowledge Bases). *IEEE Transaction on Knowledge and Data Engineering*, vol. 10(1), pp. 76-90, 1998
15. Lin, J., Mendelzon, A.: Merging databases under constraints. *International Journal of Cooperative information Systems* vol. 7(1), pp. 55-76, 1998.
16. Hansson S.: Kernel contraction. *Journal of Symbolic Logic*, vol. 59(3), 1994.
17. Hansson S.: Semi-revision. *Journal of Applied Non-Classical Logic*, pp. 151-175, 1997.
18. Hansson S.: A survey of non-prioritized belief revision. *Erkenntnis*, vol. 50, pp. 413-427, 1999.
19. Nelbel, B.: Syntax-Based Approaches to Belief Revision. In *Belief Revision*, P. Grdenfors (eds.), Cambridge Tracts in Theoretical Computer Science 29, Cambridge University Press, Cambridge, UK, pp. 52-88, 1992.
20. Qi, G., Liu, W., Glass, D. H.: A Split-Combination Method for Merging Inconsistent Possibilistic Knowledge Bases. In *Proc. KR'04*, pp. 348-356, 2004.
21. Qi, G., Liu, W., Glass, D.: Combining Individually Inconsistent Prioritized Knowledge Bases. In *Proc. of NMR'04*, pp. 342-349, 2004.
22. Qi, G., Liu, W., Bell, David.A.: Measureing conflict and agreement in a prioritized knowledge base. In *Proc. of IJCAI'05*, to appear, 2005.
23. Revesz, P. Z.: On the semantics of arbitration. *International Journal of Algebra and Computation*, vol. 7(2), pp. 133-160, 1997.
24. Williams, M. A.: A practical approach to belief revision: reason-based change. In *Proc. of KR'96*, pp. 412-421, 1996.