Merging Stratified Knowledge Bases under Constraints

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Abstract

In this paper, we propose a family of operators for merging stratified knowledge bases under integrity constraints. The operators are defined in a model-theoretic way. Our merging operators can be used to merge stratified knowledge bases where no numerical information is available. Furthermore, the original knowledge bases to be merged can be individually inconsistent. Both logical properties and computational complexity issues of the operators are studied.

Introduction

Fusion of information coming from different sources is crucial to build intelligent systems (Bloch and Hunter 2001). In classical logic, this problem is often called belief merging, which defines the beliefs (resp. goals) of a group of agents from their individual beliefs (resp. goals). There are mainly two families of belief merging operators: model-based ones which select some interpretations that are the "closest" to the original bases (Revesz 1997; Konieczny and Pino Pérez 2002; Liberatore and Schaerf 1998; Everaere, Konieczny, and Marquis 2005) and formula-based ones which pick some formulae in the union of the original bases (Baral, Kraus, and Minker 1991).

It is well-known that priority or preference (either implicit or explicit) plays an important role in many Artificial Intelligence areas, such as inconsistency handling (Benferhat et al. 1993), belief revision (Gärdenfors 1988), belief merging (Benferhat et al. 2002). When explicit priority or preference information is available, a knowledge base is stratified or ranked. In that case, the merging operators in classical logic are not appropriate to merge those knowledge bases because the priority information is not used. Merging of stratified knowledge bases is often handled in the framework of possibilistic logic (Dubois, Lang, and Prade 1994) or ordinal conditional function (Spohn 1988). The merging methods are usually based on the commensurability assumption, that is, all knowledge bases share a common scale (usually ordinal scales such as [0,1]) to order their beliefs. However, this assumption is too strong in practice-we may only have knowledge bases with a total pre-order relation on their elements. Furthermore, different agents may use different strategies to order their beliefs or interpretations. Even a single agent may have different ways of modeling her preferences for different aspects of a problem (Brewka 2004). Without the commensurability assumption, the previous merging methods are hard to apply. In addition, it is unclear how merging methods in possibilistic logic and ordinal conditional function framework can be defined in a model-theoretical way.

In this paper, we propose a family of operators for merging stratified knowledge bases under integrity constraints. The operators are defined in a model-theoretic way. We assume that each stratified knowledge base is assigned to an ordering strategy. First, for each stratified knowledge base K, the set Ω of possible worlds is stratified as $\Omega_{K,X}$ according to its ordering strategy X. In this way, a possible world has a priority level with regard to each knowledge base which is its priority level in $\Omega_{K,X}$. Second, each possible world or interpretation is associated with a list of *priority levels* in all the original knowledge bases. Then a possible world is viewed as a model of the resulting knowledge base of merging if it is a model of the formula representing the integrity constraint and it is minimal among models of the integrity constraint $w.r.t$ the lexicographical order induced by the natural order.

The main contributions of this paper are summarized as follows: First, we define our merging operators in a model-theoretic way. When the original knowledge bases are flat, i.e. there is no rank between their elements, some of our operators are reduced to existing classical merging operators. Second, the commensurability assumption is not necessary for our operators. Moreover, each knowledge base can have its own ordering strategy. By considering the pros and cons of different ordering strategies, we can deal with merging of knowledge bases in a more flexible way. Finally, we generalize the set of postulates proposed in (Konieczny and Pino Pérez 2002) for merging operators applied to stratified knowledge bases and discuss the logical properties of our operators based on these postulates.

This paper is organized as follows. Some preliminaries are introduced in Section 2. In Section 3, we consider the preference representation of stratified knowledge bases. The Δ^{PLMIN} operators are proposed in Section 4. Section 5 analyzes the computational complexity of our merging operators. We then study the logical properties of our merging operators in Section 6. Section 7 is devoted to discussing related work. Finally, we conclude the paper in Section 8.

Preliminaries

Classical logic: In this paper, we consider a propositional language $\mathcal{L}_{PS}$ from a finite set PS of propositional symbols. The classical consequence relation is denoted as $\vdash$. An interpretation (or world) is a total function from PS to $\{0, 1\}$, denoted by a bit vector whenever a strict total order on PS is specified. Ω is the set of all possible interpretations. An interpretation w is a model of a formula ϕ iff $w(\phi) = 1$. $p, q, r, ...$ represent atoms in PS. We denote formulae in $\mathcal{L}_{PS}$ by $\phi, \psi, \gamma, ...$ For each formula ϕ, we use $M(\phi)$ to denote its set of models. A *classical knowledge base K* is a finite set of propositional formulae (we can also identify K with the conjunction of its elements). K is consistent iff there exists an interpretation w such that $w(\phi) = true$ for all $\phi \in K$. A knowledge *profile E* is a multi-set of knowledge bases, i.e. $E = \{K_1, ..., K_n\}$, where K_i may be identical to K_j for $i \neq j$. Let $\bigcup(E) = \cup_{i=1}^{n} K_i$. Two knowledge profiles E_1 and E_2 are equivalent, denoted $E_1 \equiv E_2$ iff there exists a bijection f between E_1 and E_2 such that for each $K \in E_1$, $f(K) \equiv K$.

Stratified knowledge base: A *stratified* knowledge base, sometimes also called ranked knowledge base (Brewka 2004) or prioritized knowledge base (Benferhat et al. 1993), is a set K of (finite) propositional formulas together with a total preorder $\leq$ on K (a preorder is a transitive and reflexive relation, and $\leq$ is a total preorder if either $\phi \leq \psi$ or $\psi \leq \phi$ holds for any $\phi, \psi \in K$)[1]. Intuitively, if $\phi \leq \psi$, then ϕ is considered to be less important than ψ. K can be equivalently defined as a sequence $K = (S_1, ..., S_n)$, where each S_i ($i = 1, ..., n$) is a non-empty set which contains all the maximal elements of $K \setminus (\cup_{j=1}^{i-1} S_j)$ w.r.t $\leq$, i.e. $S_i = \{\phi \in K \setminus (\cup_{j=1}^{i-1} S_j) : \forall \psi \in K \setminus (\cup_{j=1}^{i-1} S_j), \psi \leq \phi\}$. Each subset S_i is called a stratum of K and i the priority level of each formula of S_i. It is clear that each formula in S_i is more reliable than formulas of the stratum S_j for $j > i$. Therefore, the lower the stratum, the higher the priority level of a formula in it. A stratified knowledge profile (SKP) E is a multi-set of stratified knowledge bases. Given a stratified knowledge base $K = (S_1, ..., S_n)$, the i-cut of K is defined as $K_{\geq i} = S_1 \cup ... \cup S_i$, for $i \in \{1, ..., n\}$. A subbase A of K is also stratified, that is, $A = (A_1, ..., A_n)$ such that $A_i \subseteq S_i$, $i = 1, ..., n$. Two SKPs E_1 and E_2 are equivalent, denoted $E_1 \equiv_s E_2$ iff there exists a bijection f between E_1 and E_2 such that for each $K = (S_1, ..., S_l) \in E_1$, $f(K) = (S'_1, ..., S'_l)$ and $S_i \equiv S'_i$ for all $i \in \{1, ..., l\}$.

Preference Representation of Stratified Knowledge Base

Ordering strategies

Given a stratified knowledge base, we can define some total pre-orders on Ω.

[1]For simplicity, we use K to denote a stratified knowledge base and ignore the total preorder $\leq$.

- **best out ordering** (Benferhat et al. 1993): let $r_{BO}(\omega) = min\{i : \omega \not\models S_i\}$, for $\omega \in \Omega$. By convention, we have $min\emptyset = +\infty$. Then the best out ordering $\preceq_{bo}$ on Ω is defined as: $\omega \preceq_{bo} \omega'$ iff $r_{BO}(\omega) \geq r_{BO}(\omega')$
- **maxsat ordering** (Brewka 2004): let $r_{MO}(\omega) = min\{i : \omega \models S_i\}$, for $\omega \in \Omega$. Then the maxsat ordering $\preceq_{maxsat}$ on Ω is defined as: $\omega \preceq_{maxsat} \omega'$ iff $r_{MO}(\omega) \leq r_{MO}(\omega')$
- **leximin ordering** (Benferhat et al. 1993): let $K^i(\omega) = \{\phi \in S_i : \omega \models \phi\}$. Then the leximin ordering $\preceq_{leximin}$ on Ω is defined as:

  $\omega \preceq_{leximin} \omega'$ iff $|K^i(\omega)| = |K^i(\omega')|$ for all i, or there is an i such that $|K^i(\omega')| < |K^i(\omega)|$, and for all $j < i$: $|K^j(\omega)| = |K^j(\omega')|$, where $|K_i|$ denote the cardinality of the sets K_i.

Given a preorder $\preceq$ on Ω, as usual, the associated strict partial order is defined by $\omega \prec \omega'$ iff $\omega \preceq \omega'$ and not $\omega' \preceq \omega$. An ordering $\preceq_X$ is more *specific* than another $\preceq_{X'}$ iff $\omega \prec_{X'} \omega'$ implies $\omega \prec_X \omega'$. The total preorders on Ω defined above are not independent of each other.

Proposition 1 *(Brewka 2004) Let $\omega, \omega' \in \Omega$, K a stratified knowledge base. The following relationships hold:*
(1) $\omega \prec_{bo} \omega'$ implies $\omega \prec_{leximin} \omega'$;
(2) $\omega \prec_{bo} \omega'$ implies $\omega \preceq_{maxsat} \omega'$ and $\omega \prec_{maxsat} \omega'$ implies $\omega \preceq_{bo} \omega'$

A new ordering strategy

We now define a new ordering strategy by considering the "distance" between an interpretation and a knowledge base.

Definition 1 *(Everaere, Konieczny, and Marquis 2005) A pseudo-distance between interpretations is a total function d from $\Omega \times \Omega$ to N such that for every ω_1, $\omega_2 \in \Omega$: (1) $d(\omega_1, \omega_2) = d(\omega_2, \omega_1)$; and (2) $d(\omega_1, \omega_2) = 0$ if and only if $\omega_1 = \omega_2$.*

A "distance " between an interpretation ω and a knowledge base S can then be defined as $d(\omega, S) = min_{\omega' \models S} d(\omega, \omega')$. When S is inconsistent, $d(\omega, S) = +\infty$. That is, all the possible worlds have the same distance with an inconsistent knowledge base. Two common examples of such distances are the *drastic distance* d_D and the *Dalal distance* d_H, where $d_D(\omega_1, \omega_2) = 0$ when $\omega_1 = \omega_2$ and 1 otherwise, and $d_H(\omega_1, \omega_2)$ is the Hamming distance between ω_1 and ω_2.

Definition 2 *The distance-based ordering $\preceq_d$ on Ω is defined as:*
$\omega \preceq_d \omega'$ iff $d(\omega, S_i) = d(\omega', S_i)$ for all i, or there is an i such that $d(\omega, S_i) < d(\omega', S_i)$, and for all $j < i$: $d(\omega, S_j) = d(\omega', S_j)$.

It is clear that the distance-based orderings are total preorders on Ω. Suppose $d = d_H$, the ordering $\preceq_{d_H}$ is equivalent to the total preorder $\leq_{K,Lex}$ which is defined to characterize the minimal change of a revision operator in (Qi, Liu, and Bell 2005).

Proposition 2 *Let $\omega, \omega' \in \Omega$, and K be a stratified knowledge base. Suppose $d = d_D$ or d_H, then we have:*
(1) $\omega \preceq_d \omega'$ implies $\omega \preceq_{bo} \omega'$ and $\omega \preceq_d \omega'$ implies $\omega \preceq_{maxsat} \omega'$; (2) $\omega \prec_{bo} \omega'$ implies $\omega \prec_d \omega'$

All proofs of this paper can be found in the accompanying technical report (Qi, Liu, and Bell 2006).

Given a stratified knowledge base K, Ω can be stratified with regard to the total preorder $\preceq$ on it obtained by an ordering strategy X as $\Omega_{K,X} = (\Omega_1, ..., \Omega_m)$ in the same way as stratifying a knowledge base. For two interpretations ω_1, ω_2, if $\omega_1 \in \Omega_i$ and $\omega_2 \in \Omega_j$, where $i < j$, then ω_1 is preferred to ω_2. We use $l_{K,X}(\omega)$ to denote the priority level of the stratum where ω belongs to, i.e. if $\omega \in \Omega_i$, then $l_{K,X}(\omega) = i$.

Δ^{PLMIN} Operators

Definition

We first introduce the lexicographical ordering.

Definition 3 *(Moulin 1988) Given two lists of numbers $\vec{a} = (a_1, ..., a_n)$ and $\vec{b} = (b_1, ..., b_n)$, where a_i and b_j are integers. Let σ be a permutation on $\{1, ..., n\}$ such that $a_{\sigma(i)} \leq a_{\sigma(i+1)}$ and $b_{\sigma(i)} \leq b_{\sigma(i+1)}$ for all i. The lexicographical ordering $\leq_{lex}$ between $\vec{a}$ and $\vec{b}$ is defined as:*

$\vec{a} \leq_{lex} \vec{b}$ if and only if $a_i = b_i$ for all i or $\exists i$ such that $a_{\sigma(i)} < b_{\sigma(i)}$ and $a_{\sigma(j)} = b_{\sigma(j)}$ for all $j < i$.

Our preference representation and lexicographical ordering based (PLMIN for short) merging operator is defined as follows.

Definition 4 *Let $E = \{K_1, ..., K_n\}$ be a multi-set of stratified knowledge bases, where $K_i = \{S_{i1}, ..., S_{im_i}\}$, and μ be an integrity constraint. Suppose X_i ($i = 1, ..., n$) are ordering strategies attached to K_i. Let $\mathbf{X} = (X_1, ..., X_n)$. Let $\preceq_{K_i,X_i}$ be the total preorder on Ω induced by the ordering strategy X_i. For each interpretation ω, we can associate with it a list of numbers $\vec{l}_E(\omega) = (l_{K_1,X_1}(\omega), ..., l_{K_n,X_n}(\omega))$, where $l_{K_i,X_i}(\omega)$ is the priority level of the stratum of Ω_{K_i,X_i} where ω belongs to. The resulting knowledge base of PLMIN merging operator, denoted by $\Delta_\mu^{PLMIN,\mathbf{X}}(E)$, is defined in a model-theoretic way as follows:*

$\omega \in M(\Delta_\mu^{PLMIN,\mathbf{X}}(E))$ iff $\omega \in M(\mu)$ and $\forall \omega' \in M(\mu)$, $\vec{l}_E(\omega) \leq_{lex} \vec{l}(\omega')$.

In Definition 4, each possible world is associated with a list of numbers consisting of the priority levels of the strata of Ω_{K_i}. Then any two possible worlds can be compared $w.r.t$ the lexicographical preference defined by Definition 3. The models of the result of the $\Delta_\mu^{PLMIN,\mathbf{X}}$ merging operator is the models of μ that are minimal $w.r.t$ the lexicographic preference. In our definition, different stratified knowledge bases may have different ordering strategies. That is, each agent can choose her own strategy to order interpretations.

Example 1 *Let $E = \{K_1, K_2, K_3\}$ be a SKP consisting of three stratified knowledge bases, where*

- *$K_1 = \{S_{11}, S_{12}, S_{13}\}$, where $S_{11} = \{p_1 \vee p_2, p_3\}$, $S_{12} = \{\neg p_1, \neg p_2, p_2 \vee \neg p_3, p_4\}$, $S_{13} = \{\neg p_3 \vee \neg p_4\}$*
- *$K_2 = \{S_{21}, S_{22}\}$, where $S_{21} = \{p_1, p_2 \vee p_3\}$ and $S_{22} = \{\neg p_2, p_4\}$*
- *$K_3 = \{S_{31}, S_{32}\}$, where $S_{31} = \{p_1, p_3\}$ and $S_{32} = \{p_2\}$.*

The integrity constraint is $\mu = \{\neg p_1 \vee p_2\}$. The set of models of μ is $M(\mu) = \{\omega_1 = 0111, \omega_2 = 0101, \omega_3 = 0110, \omega_4 = 0100, \omega_5 = 0011, \omega_6 = 0001, \omega_7 = 0010, \omega_8 = 0000, \omega_9 = 1111, \omega_{10} = 1101, \omega_{11} = 1110, \omega_{12} = 1100\}$. We denote each model by a bit vector consisting of truth values of (p_1, p_2, p_3, p_4). For example, $\omega_1 = 0111$ means that the truth value of p_1 is 0 and the truth values of other atoms are all 1. Let $\mathbf{X} = \{X_1, X_2, X_3\}$, where $X_1 = X_2 = bo$ and $X_3 = d_H$. That is, the best out ordering strategy is chosen for both K_1 and K_2, whilst the Dalal distance-based ordering is chosen for K_3. The computations are given in Table 1 below.

ω	K_1	K_2	K_3	$\vec{l}_E(\omega)$
0111	1	3	3	(1,3,3)
0101	2	3	5	(2,3,5)
0110	1	3	3	(1,3,3)
0100	2	3	5	(2,3,5)
0011	2	3	4	(2,3,4)
0001	2	3	6	(2,3,6)
0010	2	3	4	(2,3,4)
0000	2	3	6	(2,3,6)
1111	**1**	**2**	**1**	**(1,2,1)**
1101	2	2	3	(2,2,3)
1110	**1**	**2**	**1**	**(1,2,1)**
1100	2	2	3	(2,2,3)

Table 1: $\Delta_\mu^{PLMIN,\mathbf{X}}$ operator

In Table 1, the column corresponding to K_i gives the priority levels of strata of Ω_{K_i} where ω_i belongs to (Ω is stratified by an ordering strategy induced by K_i). The column corresponding to $\vec{l}_E(\omega)$ gives the lists of numbers of the priority levels of possible worlds. We explain how to obtain the column corresponding to K_2 (other columns can be obtained similarly). Let $\omega_{13} = 1011$, $\omega_{14} = 1001$, $\omega_{15} = 1010$ and $\omega_{16} = 1000$. Since $r_{BO}(\omega_i) = 1$ for all $1 \leq i \leq 8$, $r_{BO}(\omega_i) = 2$ for $9 \leq i \leq 12$ and $14 \leq i \leq 16$, $r_{BO}(\omega_{13}) = +\infty$, we have $\Omega_{K_2,bo} = (\{\omega_{13}\}, \{\omega_9, ..., \omega_{12}, \omega_{14}, ..., \omega_{16}\}, \{\omega_1, ..., \omega_8\})$. So $l_{K_2,bo}(\omega_i) = 3$ for $1 \leq i \leq 8$ and $l_{K_2,bo}(\omega_i) = 2$ for $9 \leq i \leq 12$. By Def. 4, it is easy to see that ω_9 and ω_{11} are two minimal possible worlds in Table 1. So $M(\Delta_\mu^{PLMIN,\mathbf{X}}(E)) = \{1111, 1110\}$. That is, $\Delta_\mu^{PLMIN,\mathbf{X}}(E) = p_1 \wedge p_2 \wedge p_3$.

The following proposition states relationships between different Δ^{PLMIN} operators when considering different ordering strategies.

Proposition 3 *Let $E = \{K_1, ..., K_n\}$ be a SKP, and μ be the integrity constraint. Let $\mathbf{X_1} = \{X_1, ..., X_n\}$ and $\mathbf{X_2} = \{X'_1, ..., X'_n\}$ be two vectors of ordering strategies, where both X_i and X'_i are ordering strategies for K_i. Suppose $\preceq_{X_i}$ is more specific than $\preceq_{X'_i}$, for all i, where $X_i \in \mathbf{X_1}$ and $X'_i \in \mathbf{X_2}$, then $\Delta_\mu^{PLMIN,\mathbf{X_2}}(E) \models \Delta_\mu^{PLMIN,\mathbf{X_1}}(E)$.*

Proposition 3 shows that the operator with regard to the set of more specific ordering strategies can result in a knowledge base which has stronger inferential power. By Proposition 2 and 3, we have the following result: Suppose $X_i = bo$ and $X'_i = d$ for all i, then $\Delta_\mu^{PLMIN,\mathbf{X_2}}(E) \models \Delta_\mu^{PLMIN,\mathbf{X_1}}(E)$.

Let us go back to Example 1.

Example 2 *(continue Example 1) Suppose* $\mathbf{X}' = \{X_1', X_2', X_3'\}$*, where* $X_1' = bo$*,* $X_2' = X_3' = d_H$*. The computations are given in Table 2 below.*

ω	K_1	K_2	K_3	$\vec{l}_E(\omega)$
0111	1	5	3	(1,5,3)
0101	2	5	5	(2,5,5)
0110	1	6	3	(1,6,3)
0100	2	6	5	(2,6,5)
0011	2	4	4	(2,4,4)
0001	2	7	6	(2,7,6)
0010	2	5	4	(2,5,4)
0000	2	8	6	(2,8,6)
1111	**1**	**2**	**1**	**(1,2,1)**
1101	2	2	3	(2,2,3)
1110	1	3	1	(1,3,1)
1100	2	3	3	(2,3,3)

Table 2: $\Delta_\mu^{PLMIN,\mathbf{X}'}$ *operator*

According to Table 2, $\omega_9 = 1111$ *is the only minimal model in* $M(\mu)$*. So the result of merging by the* $\Delta^{PLMIN,d}$ *operator is* $M(\Delta_\mu^{PLMIN,\mathbf{X}'}(E)) = \{1111\}$*. So* $\Delta_\mu^{PLMIN,\mathbf{X}'}(E) = p_1 \wedge p_2 \wedge p_3 \wedge p_4$*. It is clear that* $M(\Delta_\mu^{PLMIN,\mathbf{X}'}(E)) \models M(\Delta_\mu^{PLMIN,\mathbf{X}}(E))$*.*

Flat case

In this section, we apply our merging operators to the classical knowledge bases. Since our merging operators are based on the ordering strategies, we need to consider the ordering strategies for classical knowledge bases.

Proposition 4 *Let K be a classical knowledge base. Suppose X is an ordering strategy, then*

1. *for $X = bo$ and $X = maxsat$, we have $\omega \preceq_X \omega'$ iff $\omega \models K$*
2. *for $X = leximin$, let $K(\omega) = \{\phi \in K : \omega \models \phi\}$, we have $\omega \preceq_X \omega'$ iff $|K(\omega)| \geq |K(\omega')|$*
3. *for $X = d$, we have $\omega \preceq_X \omega'$ iff $d(\omega, K) \leq d(\omega, K')$.*

By Proposition 4, the best out ordering and the maxsat ordering are reduced to the same ordering when knowledge base is classical. Furthermore, the leximin ordering can be used to order possible worlds when the knowledge base is inconsistent.

Proposition 5 *Let E be a knowledge profile and μ be a formula. Let $MAXCONS(E, \mu) = \{F \subseteq E : \bigcup(F) \cup \{\mu\} \not\models \bot$, and if $F \subset E' \subseteq E$, then $\bigcup(E') \cup \{\mu\} \models \bot\}$. That is, $MAXCONS(E, \mu)$ is the set of maximal subsets of E which are consistent with μ. Let $CardM(E, \mu) = \{F \in MAXCONS(E, \mu) : \nexists F' \in MAXCONS(E, \mu), |F| < |F'|\}$. Suppose $X_i = bo$ or $maxsat$ for all i, then $\Delta_\mu^{PLMIN,\mathbf{X}}(E) = \bigvee_{F \in CardM(E,\mu)} (\wedge_{\phi \in F} \phi \wedge \mu)$.*

Proposition 5 shows that the $\Delta^{PLMIN,\mathbf{X}}$ operator is equivalent to the $\triangle^{C4}$ operator defined in (Konieczny, Lang, and Marquis 2004), which selects the set of consistent subsets of $E \cup \{\mu\}$ that contain the constraints μ and that are maximal with respect to cardinality, when each knowledge base is viewed as a formula and ordering strategy of it is the *best out* strategy or *maxsat* strategy.

When $X_i = d$ for all i, the corresponding $\Delta_\mu^{PLMIN,\mathbf{X}}$ operators are similar to the $\triangle_\mu^{d,Gmin}$ operators defined as follows.

Definition 5 *(Everaere, Konieczny, and Marquis 2005) Let d be a pseudo-distance, μ an integrity constraint, $E = \{K_1, ..., K_n\}$ a profile and let ω be an interpretation. The "distance" between ω and E, denoted by $d_{d,Gmin}(\omega, E)$, is defined as the list of numbers $(d_1, ..., d_n)$ obtained by sorting in increasing order the set $\{d(\omega, K_i) : K_i \in E\}$. The models of $\triangle_\mu^{d,Gmin}(E)$ are the models of μ that are minimal $w.r.t$ the lexicographical order induced by the natural order.*

Our Δ_μ^{PLMIN} operators and the $\triangle_\mu^{d,Gmin}$ operators differ in that the lists of numbers attached to models are different. The former uses the priority levels of a model $w.r.t$ all the knowledge bases and the latter uses the distance between a model and each knowledge base.

Proposition 6 *Let $E = \{K_1, ..., K_n\}$ a profile and μ an integrity constraint. d_D is the drastic distance and $\mathbf{X} = (X_1, ..., X_n)$ is a set of ordering strategies attached to K_i ($i = 1, ..., n$), where $X_i = d_D$ for all i. Then $\Delta_\mu^{PLMIN,\mathbf{X}}(E) \equiv \triangle_\mu^{d_D,Gmin}(E)$.*

Proposition 6 shows that the $\Delta_\mu^{PLMIN,\mathbf{X}}$ operator and the $\triangle_\mu^{d_D,Gmin}$ operator are equivalent when the *drastic distance* is chosen.

Propositions 5 and 6 only consider $\triangle_\mu^{d,Gmin}$ operators where all knowledge bases have the same ordering strategy. When hybrid ordering strategies are used, we can get more operators. For example, if we use the $leximin$ ordering for those knowledge bases which are inconsistent, then our operators can be applied to merging a set of knowledge bases which may be individually inconsistent. Now let us look at an example.

Example 3 *Let $E = \{K_1, K_2\}$, where $K_1 = \{p_1 \vee p_2, p_3, \neg p_3\}$ and $K_2 = \{p_1, p_2, p_3\}$, and $\mu = \{(p_1 \vee p_3) \wedge p_2\}$. So $Mod(\mu) = \{\omega_1 = 110, \omega_2 = 111, \omega_3 = 011\}$. Let $\mathbf{X} = (X_1, X_2)$, where $X_1 = leximin$ and $X_2 = bo$ are ordering strategies of K_1 and K_2 respectively. The computations are given in Table 3 below.*

ω	K_1	K_2	$\vec{l}_E(\omega)$
110	1	2	(1,2)
111	**1**	**1**	**(1,1)**
011	1	2	(1,2)

Table 3: $\Delta_\mu^{PLMIN,\mathbf{X}}$ *operator*

According to Table 3, $\omega_2 = 111$ *is the only minimal model in* $M(\mu)$*. So* $M(\Delta_\mu^{PLMIN,\mathbf{X}}(E)) = \{111\}$*. That is,* $\Delta_\mu^{PLMIN,\mathbf{X}}(E) = p_1 \wedge p_2 \wedge p_3$*.*

Computational Complexity

We now discuss the complexity issue. First we need to consider the computational complexity of stratifying Ω from a stratified knowledge base. In (Lang 2004), two important problems for logical preference representation languages were considered. We express them as follows.

Definition 6 *Given a stratified knowledge base K and two interpretations ω and ω', the COMPARISON problem consists of determining whether $\omega \preceq_X \omega'$, where X denotes an ordering strategy. The NON-DOMINANCE problem consists of determining whether ω is* non-dominated *for $\preceq_X$, that is, there is not ω' such that $\omega' \prec_X \omega$.*

It was shown in (Lang 2004) that the NON-DOMINANCE problem is usually a hard problem, i.e **coNP**-complete. We have the following proposition on NON-DOMINANCE problem for ordering strategies in Section 3.

Proposition 7 *Let K be a stratified knowledge base. For $X = bo$, $maxsat$, or $lexmin$:*
(1) COMPARISON is in P*, where* P *denotes the class of problems decidable in deterministic polynomial time.*
*(2) NON-DOMINANCE is co***NP***-complete.*

To stratify Ω, we need to consider the problem *determining all non-dominated interpretations*, which is computational much harder than the NON-DOMINANCE problem (we believe it is Σ_2^p-hard). To simplify the computation of our merging operators, we assume that Ω is stratified from each stratified knowledge base during an off-line preprocessing stage.

Let Δ be a merging operator. The following decision problem is denoted as MERGE(Δ):

- **Input :** a 4-tuple $\langle E, \mu, \psi, \mathbf{X}\rangle$ where $E = \{K_1, ..., K_n\}$ is a SKP, μ is a formula, and ψ is a formula; $\mathbf{X} = (X_1, ..., X_n)$, where X_i is the ordering strategy attached to K_i.
- **Question :** Does $\Delta_\mu(E) \models \psi$ hold?

Proposition 8 *MERGE($\Delta^{PLMIN,\mathbf{X}}$) in Θ_2^p. Let $\mathbf{X} = (X_1, ..., X_n)$, where $X_i = bo$, $maxsat$, or $leximin$ ($i = 1, ..., n$), then MERGE($\Delta^{PLMIN,\mathbf{X}}$) is Θ_2^p-complete.*

Proposition 8 shows that the computational complexity of inference for our merging operators is located at a low level of the boolean hierarchy under an additional assumption.

Logical Properties

Many logical properties have been proposed to characterize a belief merging operator. We introduce the set of postulates proposed in (Konieczny and Pino Pérez 2002), which is used to characterize Integrity Constraints (*IC*) merging operators.

Definition 7 *Let E, E_1, E_2 be knowledge profiles, K_1, K_2 be consistent knowledge bases, and μ, μ_1, μ_2 be formulas from $\mathcal{L}_{PS}$. Δ is an IC merging operator iff it satisfies the following postulates:*
(IC0) $\Delta_\mu(E) \models \mu$
(IC1) *If μ is consistent, then $\Delta_\mu(E)$ is consistent*
(IC2) *If $\bigwedge E$ is consistent with μ, then $\Delta_\mu(E) \equiv \bigwedge E \wedge \mu$, where $\bigwedge(E) = \wedge_{K_i \in E} K_i$*
(IC3) *If $E_1 \equiv E_2$ and $\mu_1 \equiv \mu_2$, then $\Delta_{\mu_1}(E_1) \equiv \Delta_{\mu_2}(E_2)$*
(IC4) *If $K_1 \models \mu$ and $K_2 \models \mu$, then $\Delta_\mu(\{K_1, K_2\}) \wedge K_1$ is consistent iff $\Delta_\mu(\{K_1, K_2\}) \wedge K_2$ is consistent*
(IC5) $\Delta_\mu(E_1) \wedge \Delta_\mu(E_2) \models \Delta_\mu(E_1 \sqcup E_2)$
(IC6) *If $\Delta_\mu(E_1) \wedge \Delta_\mu(E_2)$ is consistent, then $\Delta_\mu(E_1 \sqcup E_2) \models \Delta_\mu(E_1) \wedge \Delta_\mu(E_2)$*
(IC7) $\Delta_{\mu_1}(E) \wedge \mu_2 \models \Delta_{\mu_1 \wedge \mu_2}(E)$
(IC8) *If $\Delta_{\mu_1}(E) \wedge \mu_2$ is consistent, then $\Delta_{\mu_1 \wedge \mu_2}(E) \models \Delta_{\mu_1}(E) \wedge \mu_2$*

The postulates are used to characterize an IC merging operator in classical logic. Detailed explanation of the above postulates can be found in (Konieczny and Pino Pérez 2002).

Some postulates in Definition 7 need to be modified if we consider merging postulates for stratified knowledge bases, i.e., $(IC2)$, $(IC3)$ should be modified as:
(IC2$'$) Let $\bigwedge E = \wedge_{K_i \in E} \wedge_{\phi_{ij} \in K_i} \phi_{ij}$. If $\bigwedge E$ is consistent with μ, then $\Delta_\mu(E) \equiv \bigwedge E \wedge \mu$
(IC3$'$) If $E_1 \equiv_s E_2$ and $\mu_1 \equiv \mu_2$, then $\Delta_{\mu_1}(E_1) \equiv \Delta_{\mu_2}(E_2)$

$(IC3')$ is stronger than $(IC3)$ because the condition of equivalence between two knowledge profiles is generalized to the condition of equivalence between two SKPs. We do not generalize $(IC4)$, the fairness postulate, which says that the result of merging of two belief bases should not give preference to one of them. This postulate is controversial (Konieczny 2004). And it is hard to be adapted in the prioritized case because a stratified knowledge base may be inconsistent and there is no unique consequence relation for a stratified knowledge base (Benferhat et al. 1993).

Proposition 9 *$\Delta_\mu^{PLMIN,\mathbf{X}}$ satisfies (IC0), (IC1), $(IC2')$, (IC5), (IC6) (IC7), (IC8). The other postulates are not satisfied in the general case.*

$(IC3')$ is not satisfied because some ordering strategies are syntax-sensitive. However, when the ordering strategies are either best-out ordering or maxsat ordering, then our merging operators satisfy all the generalized postulates.

Proposition 10 *Suppose $X_i = bo$ or $maxsat$, then $\Delta_\mu^{PLMIN,\mathbf{X}}$ satisfies (IC0), (IC1), $(IC2')$, $(IC3')$, (IC5), (IC6), (IC7), (IC8). The other postulates are not satisfied in the general case.*

Related Work

Merging of stratified knowledge bases is often handled in the framework of possibilistic logic (Dubois, Lang, and Prade 1994) or ordinal conditional function (Spohn 1988). In possibilistic logic, the merging problems are often solved by aggregating *possibility distributions*, which are mappings from Ω to a common scale such as [0,1], using some *combination modes*. Then the syntactic counterpart of these combination modes can be defined accordingly (Benferhat et al. 2002). In (Meyer, Ghose, and Chopra 2002), the merging is conducted by merging *epistemic states* which are (total) functions from the set of interpretations to **N**, the set of natural numbers. There are many other merging methods in possibilistic logic (Benferhat et al. 1999) and in ordinal conditional function framework (Benferhat et al. 2004). Our merging operators differs from previous ones in two aspects:

First, our operators are semantically defined in a model-theoretic way and others are semantically defined by distribution functions such as possibility distributions. In the flat case, our merging operators belong to model-based merging operators, and they capture some notion of minimal change. Whilst other merging operators are usually syntax-based ones in the flat case.

Second, most of previous merging operators are based on the commensurability assumption. In (Benferhat et al. 1999), a merging approach for stratified knowledge base is proposed which drops the commensurability assumption. However, their approach is based on the assumption that there is an ordering relation between two stratified knowledge bases K_1 and K_2, i.e. K_1 has priority over K_2. In contrast, our merging operators do not require any of above assumptions and are flexible enough to merge knowledge bases which are stratified by a total pre-ordering on their elements. So our merging operators are more general and practical than other methods.

This work is also related to the logical preference description language (LPD) in (Brewka 2004). The language LPD uses binary operators $\vee$, $\wedge$ and $>$ to connect two (or more) *basic orderings* and get more complex orderings. In contrast, when defining our merging operators, we use an adaptive method which is based on a lexicographical preference to combine orderings assigned to original knowledge bases.

Conclusions and Further Work

In this paper, we have proposed a family of model-theoretic operators to merge stratified knowledge bases under integrity constraints. In the flat case, some of our operators are reduced to existing merging operators. The computational complexity of our merging operators has been analyzed. Under an additional assumption, the computation of Δ^{PLMIN} is equivalent to that of Δ^{GMIN} in (Everaere, Konieczny, and Marquis 2005). Finally, we have generalized the set of postulates defined in (Konieczny and Pino Pérez 2002) and shown that our operators satisfy most of the generalized postulates.

There are several issues remaining and these require further work. First, we have applied our merging operators to classical bases and have obtained some interesting results. However, to have a thorough evaluation of our operators in the flat case, we need to consider other important criteria to compare operators, such as strategy-proofness and discriminating power. Second, we revised the set of postulates defined in (Konieczny and Pino Pérez 2002). However, the revision is a simple extension of existing postulates. More postulates will be explored in the future.

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