A General Model for Epistemic State Revision using Plausibility Measures

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Abstract. In this paper, we present a general revision model on epistemic states based on plausibility measures proposed by Friedman and Halpern. We propose our revision strategy and give some desirable properties, e.g., the reversible and commutative properties. Moreover, we develop a notion called *plausibility kinematics* and show that our revision strategy follows plausibility kinematics. Furthermore, we prove that the revision following plausibility kinematics satisfies the principle of minimal change based on some distance measures. Finally, we discuss a revision operator defined for plausibility functions and its relationship with iterated belief revision proposed by Darwiche and Pearl. We show that the revision operator satisfies all the DP postulates when it is Max-Additive.

1 Introduction

Belief revision [AGM85, KM91, DP97] is a significant subarea of artificial intelligence and philosophy. It depicts the process that an agent revises its beliefs upon receiving new evidence, under the assumption that an agent always takes the new information as the most reliable one and uses it to revise its current beliefs to reach a new consistent set of beliefs.

In recent years, many researchers realized that epistemic states (not just their belief sets) should play an important, even fundamental role in iterated belief revision [DP97, B+00, NPP03, B+05, JT07]. These papers are concerned with the logic of iterated revision with the integration of epistemic states. More precisely, an agent's current beliefs are modeled with epistemic states and new evidence is in the form of propositional logic formula. In contrast to the above approaches to epistemic state revision derived from the AGM revision framework in logics, epistemic state revision has also been studied in numerical settings. In [Spo88], ordinal conditional functions (OCFs, also known as ranking functions [Hal03]) are introduced to render the dynamics of the change of epistemic states (i.e., epistemic state revision). In [DP93], a counterpart in possibility theory was proposed by Dubois and Prade.

In this paper, we present a generalized model for the dynamics (strategies) of epistemic state revision under the framework of plausibility measures introduced by Friedman and Halpern [FH95, Hal01], which takes OCFs and possibility measures as its special cases. We also investigate if our revision strategy is optimal such that it satisfies the principle of minimal change. Moreover, we want our general model satisfying all the iterated belief revision postulates, e.g., DP postulates [DP97]. We prove that it requires the plausibility measure to be Max-Additive in order to satisfy DP postulates.

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The remainder of this paper is organized as follows. Section 2 provides some preliminary knowledge of OCFs, possibility functions, and plausibility measures. Section 3 introduces our general revision model and its properties. We also show that the revision model satisfies the principle of minimal change. In section 4, we use the iterated belief revision postulates in [DP97] to verify our model. Finally, in Section 5, we draw a conclusion of the paper.

2 Preliminaries

2.1 Ordinal Conditional Functions

An **ordinal conditional function** [Spo88], also known as a **ranking function** [Hal03] or a **kappa**-function, commonly denoted as κ, is a function from a set of possible worlds, W, to the set of ordinal numbers. Function κ is normalized (consistent) if there exists at least one possible world s.t. $\kappa(w) = 0$. Value $\kappa(w)$ is understood as the degree of **disbelief** of world w. So the smaller the value, the more plausible the world is. The ranking value of a set A (i.e. a proposition μ_A) is defined as:

$$\kappa(A) = min_{w \in A} \kappa(w)$$

or $$\kappa(\mu) = min_{w \models \mu} \kappa(w), Mod(\mu) = A.$$

The conditioning of ordinal conditional function is defined as:

$$\kappa(B|A) = min_{w \in A \cap B}(\kappa(w)) - \kappa(A) = \kappa(A \cap B) - \kappa(A).$$

Note that in [Spo88], $\kappa(\emptyset) = \infty$. So when $A \cap B = \emptyset$, $\kappa(B|A) = \infty$.

In [Spo88], the (A, α)-conditionalization, also commonly considered as (A, α)-revision, is proposed as follows. Let an agent's current belief be represented by an OCF κ, and let new evidence concerning event A be given as $\kappa^{'}(A) = 0$ and $\kappa^{'}(\overline{A}) = \alpha$ (where $\overline{A} = W \setminus A$), then the revised κ (by κ') is defined as:

$$\kappa_r(w) = \begin{cases} \kappa(w|A) & \text{for } w \in A \\ \alpha + \kappa(w|\overline{A}) & \text{for } w \in \overline{A} \end{cases} \quad (1)$$

2.2 Possibility Theory

Semantically, a possibility distribution π is a mapping from W to $[0, 1]$. It induces a possibility measure $\Pi : 2^W \rightarrow [0, 1]$ and a necessity measure $N : 2^W \rightarrow [0, 1]$ as follows:

$$\Pi(A) = max_{w \in A} \pi(w) \; and \; N(A) = 1 - \Pi(\overline{A}).$$

$\Pi(A)$ estimates the degree an agent believes the true world can be in A while $N(A)$ estimates the degree the agent believes the true world should be necessarily in A.

There are several conditioning methods in possibility theory, and we adopt the following one in this paper [DP93].

$$\Pi(B|A) \stackrel{def}{=} \frac{\Pi(B \cap A)}{\Pi(A)} \tag{2}$$

A counterpart of Spohn's (A, α)-conditionalization was suggested in [DP93] in possibility theory such that if new evidence suggests that $\Pi^{'}(A) = 1$ and $\Pi^{'}(\overline{A}) = 1 - \alpha$ (which implies that $N^{'}(A) = \alpha$), then the belief change of an agent's current belief π can take the following form

$$\pi_r(w) = \begin{cases} \pi(w|A) & \text{for } w \in A \\ (1-\alpha)\pi(w|\overline{A}) & \text{for } w \in \overline{A} \end{cases} \tag{3}$$

where $\pi(w|A) = \pi(w)/\Pi(A)$ which can be derived from Equation 2 with B being a singleton, i.e., $B = \{w\}$.

2.3 Plausibility Measure

Definition 1 *[FH95, Hal03] A* **plausibility space** *is a tuple $S = (W, \mathscr{F}, D, Pl)$, where W is a set of possible worlds, $\mathscr{F}$ is an algebra over W*[2]*, D is a domain of plausibility values partially ordered by a relation $\leq_D$, and Pl maps sets in $\mathscr{F}$ to D. D is assumed to contain two special elements, $\top$ and $\bot$, such that $\bot \leq_D d \leq_D \top$ for all $d \in D$. Besides, the* **plausibility measure** *Pl should satisfy the following conditions:*

Pl1 $Pl(\emptyset) = \bot$
Pl2 $Pl(W) = \top$
Pl3 *If $U \subseteq V$, then $Pl(U) \leq_D Pl(V)$*

For example, if Pl is reduced to a probability measure, then $\bot = 0$, $\top = 1$ and $\leq_D$ is $\leq$. If Pl is reduced to an OCF, then $\bot = +\infty$, $\top = 0$ and $\leq_D$ is $\geq$, and if $\leq$ is reduced to a possibility measure, then $\bot = 0$, $\top = 1$ and $\leq_D$ is $\leq$.

In [Hal01], a plausibility measure Pl is **additive** with respect to $\oplus$ such that $Pl(U \cup V) = Pl(U) \oplus Pl(V)$ for disjoint $U, V \in \mathscr{F}$ where $\oplus$ is a mapping from $D \times D$ to D. The conditioning of Pl on A, denoted as $Pl(\cdot|A)$, is defined as satisfying

CPl1 $Pl(\emptyset|A) = \bot$
CPl2 $Pl(W|A) = \top$
CPl3 If $U \subseteq V$, then $Pl(U|A) \leq_D Pl(V|A)$
CPl4 $Pl(U|A) = Pl(U \cap A|A)$

Furthermore, Pl is said **algebraic** if it satisfies the following:

Alg1 If $U \cap V = \emptyset$, then $Pl(U \cup V|V^{'}) = Pl(U|V^{'}) \oplus Pl(V|V^{'})$
Alg2 $Pl(U \cap V|V^{'}) = Pl(U|V \cap V^{'}) \otimes Pl(V|V^{'})$
Alg3 $\otimes$ distributes over $\oplus$; more precisely, $a \otimes (b_1 \oplus b_2 \oplus \ldots \oplus b_n) = (a \otimes b_1) \oplus (a \otimes b_2) \oplus \ldots \oplus (a \otimes b_n)$
Alg4 $a \otimes c \leq_D b \otimes c$ and $c \neq \bot$ implies $a \leq_D b$

where $\otimes$ is a mapping from $D \times D$ to D.

To put operators $\oplus$ and $\otimes$ into perspective with respect to probability measures, OCFs, and possibility measures, we have $(\oplus = +, \otimes = \times)$ for a probability measure Pr, $(\oplus = min, \otimes = +)$ for an OCF κ, and $(\oplus = max, \otimes = \times)$ for a possibility measure Π respectively.

[2] An algebra over W is a set of subsets of W closed under complementation and union.

Proposition 1 *Let $d \in D$, we have $d \otimes \top = \top \otimes d = d$.*

To make the subsequent discussion easier, we have the following: let A be any set, for any binary relation $\leq$ over $A \times A$, $<$ is defined as $a < b$ iff $a \leq b$ and $b \not\leq a$, and $=$ is defined as $a = b$ iff $a \leq b$ and $b \leq a$, for $a, b \in A$.

3 Epistemic State Revision by Plausibility Measures

Here we present a revision model for epistemic state change using plausibility measures. This model is general enough to subsume the conditionalization of ordinal conditional functions, Jeffrey's rule of probability updating, and the revision operator (Equation 3) in possibility theory introduced above.

For this purpose, we need to define some simple and rational properties for operator $\otimes$ mentioned in the last section.

Definition 2 *Let $S = (W, \mathscr{F}, D, Pl)$ be a plausibility space, a, b, c be any elements in D and $\otimes$ be a mapping from $D \times D$ to D, then $\otimes$ is called*

reversible *iff there exists a mapping $\otimes^{-1}$ such that $a \otimes^{-1} b \otimes b = a$ and $a \otimes b \otimes^{-1} b = a$ for $b \neq \bot$.*
commutative *iff $a \otimes b = b \otimes a$.*
associative *iff $a \otimes (b \otimes c) = a \otimes b \otimes c$*
equal-ranking *iff $a \otimes b \otimes^{-1} c = a \otimes^{-1} c \otimes b$ for $c \neq \bot$.*
right-sign-keeping *iff $a \otimes c <_D b \otimes c$ for $a <_D b$.*
left-sign-keeping *iff $c \otimes a <_D c \otimes b$ for $a <_D b$.*
sign-keeping *iff $\otimes$ is both right-sign-keeping and left-sign-keeping.*

Property **equal-ranking** says that an operation $\otimes$ and its reversing operation $\otimes^{-1}$ have the same level of operation grade, such as, '+' and its reverse '-' have the same level of arithmetic calculation grade and they are a grade lower than '$\times$' and '/'.

Note that if $\otimes$ is reversible, then by setting $V' = W$ in **Alg2**, we obtain a conditional plausibility as follows.

$$Pl(U|V) = Pl(V \cap U) \otimes^{-1} Pl(V). \tag{4}$$

The reason we need to have both the right-sign-keeping and left-sign-keeping conditions is that some operators may not be associative, so these two conditions are not totally equivalent.

Proposition 2 *Let $S = (W, \mathscr{F}, D, Pl)$ be a plausibility space and $\otimes$ be a reversible and right-sign-keeping mapping from $D \times D$ to D, then $\otimes^{-1}$ is right-sign-keeping.*

Note that if $\otimes$ is commutative, then $\otimes$ is right-sign-keeping iff $\otimes$ is left-sign-keeping. But we still differentiate the two situations as there may be non-commutative operators, e.g., $\otimes^{-1}$.

Definition 3 *Let $S = (W, \mathscr{F}, D, Pl)$ be a plausibility space and $\otimes$ be a mapping from $D \times D$ to D, then $\otimes$ is called a* **rational mapping** *iff it satisfies* reversible, commutative, associative, equal-ranking, and sign-keeping.

Proposition 3 *Let $S = (W, \mathscr{F}, D, Pl)$ be a plausibility space and $\otimes$ be a rational mapping from $D \times D$ to D, then for any $a, b, c, d \in D$ and $b, c \neq \bot$, we have*

1. $a \otimes^{-1} b \otimes^{-1} c = a \otimes^{-1} c \otimes^{-1} b$,
2. $a \otimes (d \otimes^{-1} c) = a \otimes d \otimes^{-1} c$,
3. $b \otimes^{-1} b = \top$,

In fact, when probability functions, OCFs and possibility functions, are viewed as plausibility functions, the corresponding $\otimes$s (which are '+', min, and max respectively) are indeed rational mappings. More formally, we have the following lemma.

Lemma 1 *(Part of this lemma can be found in [Hal01]) Let Pr be a probability function, κ be an OCF, and Π be a possibility function. When considered as a plausibility function Pl, it satisfies the followings:*

1. *Pl is additive with respect to the corresponding $\oplus$.*
2. *The conditioning $Pl_A(B)$ can be written as $Pl(A \cap B) \otimes^{-1} Pl(A)$ and Pl_A is also a probability function (resp. OCF κ, possibility measure Π) if the original Pl is Pr (resp. κ, Π).*
3. *$\otimes$ is a rational mapping (Def 3).*
4. *$\otimes$ distributes over $\oplus$ (**Alg1**).*

We define the revision model by plausibility measures as follows.

Definition 4 *Let $S = (W, 2^W, D, Pl)$ be a plausibility space for the prior, and $S_e = (W, \mathscr{F}_e, D, Pl_e)$ be the plausibility space for new evidence where $\mathscr{F}_e = 2^{\{A_1,\ldots,A_n\}}$ is the powerset of a partition of W, then the revised plausibility measure, denoted as Pl_{re}, is*

$$Pl_{re}(w) = Pl_e(A_i) \otimes^{-1} Pl(A_i) \otimes Pl(w), w \in A_i, 1 \le i \le n.$$

Proposition 4 *Let $S = (W, 2^W, D, Pl)$ be a plausibility space for the prior, and $S_e = (W, \mathscr{F}_e, D, Pl_e)$ be the plausibility space for new evidence where $\mathscr{F}_e = 2^{\{A_1,\ldots,A_n\}}$, then we have*

$$Pl_{re}(A_i) = Pl_e(A_i), 1 \le i \le n.$$

This proposition shows that the above definition indeed reserves the value $Pl_e(A_i)$ from the evidence, so it satisfies the general requirement in revision that the new evidence has to be preserved.

Here are some general properties of the revision by plausibility measures.

Proposition 5 *Let $S = (W, \mathscr{F} = 2^W, D, Pl)$ be a plausibility space for the prior state and $S_{e1} = (W, \mathscr{F}_{e1}, D, Pl_{e1})$, $S_{e2} = (W, \mathscr{F}_{e2}, D, Pl_{e2})$ be two plausibility spaces for two new pieces of evidence such that $\mathscr{F}_{e1} = \mathscr{F}_{e2} = 2^{\{A_1,\ldots,A_n\}}$, then we have $(Pl_{re1})_{re2} = Pl_{re2}$.*

This proposition reveals that if two pieces of evidence are about the same event but differ on the strengthes, then the evidence arriving later will suppress the former.

When new evidence is given on $\mathscr{F}_e = 2^{\{A,\overline{A}\}}$ within a plausibility measure, the above revision is reduced to the well known (A, α)-revision with OCFs [Spo88, DP93] which is the revision when $S_e = (W, 2^{\{A,\overline{A}\}}, D, Pl_e)$ such that $Pl(A) = \top$ and $Pl(\overline{A}) = \alpha$. Thus we have

Proposition 6

$$Pl_{A,\alpha}(w) = \begin{cases} Pl(w) \otimes^{-1} Pl(A) & \text{for } w \in A, \\ \alpha \otimes^{-1} Pl(\overline{A}) \otimes Pl(w) & \text{for } w \in \overline{A}. \end{cases} \quad (5)$$

For the (A, α)-revision, we have the following corollary.

Corollary 1 *(Reversible) Let $S = (W, \mathscr{F} = 2^W, D, Pl)$ be a plausibility space and $A \in \mathscr{F} \setminus \{\emptyset, W\}$ such that $Pl(A) = \top$ and $Pl(\overline{A}) = \beta$, then we have $(Pl_{A,\alpha})_{A,\beta} = (Pl_{\overline{A},\alpha})_{A,\beta} = Pl$.*

This corollary is a direct generalization of Theorem 3 in [Spo88] for OCFs.

Definition 5 *Let $\oplus$ be a mapping from $D \times D$ to D, then $\oplus$ is called bounded-additive if and only if it follows: $\top \oplus d = d \oplus \top = \top$ for all $d \in D$.*

For convenience, if Pl is associated with a bounded-additive $\oplus$, then we simply call Pl is bounded-additive.

It is clear to see that OCFs and possibility measures are bounded-additive, but unfortunately, the probability function is not bounded-additive.

Lemma 2 *Let κ be an OCF and Π be a possibility measure. When considered as plausibility measures, they are bounded-additive.*

For bounded additive Pl, we have the following theorem.

Proposition 7 *(Commutative) Let Pl be a bounded-additive plausibility measure and A, $B \in \mathscr{A} \setminus \{\emptyset, W\}$ such that $Pl(A \cap B) = Pl(A \cap \overline{B}) = Pl(\overline{A} \cap B) = \top$, then we have $(Pl_{A,\alpha})_{B,\beta} = (Pl_{B,\beta})_{A,\alpha}$.*

In Theorem 4 [Spo88], Spohn pointed out that accumulated epistemic revision on events satisfying certain conditions ($\kappa(A \cap B) = \kappa(A \cap \overline{B}) = \kappa(\overline{A} \cap B) = 0$) should be commutative. Here we generalize the theorem to the plausibility case and give the above proposition which is the counterpart of the theorem.

The revision by Definition 4 can be equivalently rewritten as

$$Pl_{re}(w) \otimes^{-1} Pl(w) = Pl_{re}(A_i) \otimes^{-1} Pl(A_i), w \in A_i$$

It is a counterpart of so called *probability kinematics* [Jef65] in probability theory. In [CD05b], it is proved that Jeffrey's Rule and Pearl's virtual evidence method (a kind of revision on Bayesian networks) both follow probability kinematics. Hence here our revision strategy can be called **plausibility kinematics**.

We give the formal definition of *plausibility kinematics* as follows:

Definition 6 *Suppose that two plausibility measures Pl and Pl' disagree on the plausibility values they assign to a set of mutually exclusive and exhaustive events $A_1, A_2, \ldots, A_n$. The distribution Pl' is said to be obtained from Pl by* **plausibility kinematics** *on $A_1, A_2, \ldots, A_n$, iff for any $w \in A_i$, $1 \le i \le n$, we have*

$$Pl'(w) \otimes^{-1} Pl(w) = Pl'(A_i) \otimes^{-1} Pl(A_i).$$

Obviously, the revision strategy in Definition 4 shows that the revised plausibility measure is obtained from the prior plausibility measure by plausibility kinematics.

Next we prove that our revision strategy does achieve the minimal change. Namely, we show that among all revision strategies, the plausibility measure obtained by plausibility kinematics has the shortest distance to the prior plausibility measure.

First, we define a distance function which is generalized from its probability counterpart in [CD05a, CD05b].

Definition 7 *Let Pl and Pl' be two plausibility measures on 2^W, then the distance between Pl and Pl' is defined as*

$$\begin{aligned} d(Pl, Pl') &= \odot(max_w Pl'(w) \otimes^{-1} Pl(w)) \\ &- \odot(min_w Pl'(w) \otimes^{-1} Pl(w)), \end{aligned}$$

where we define here $\perp \otimes^{-1} \perp = \top$, and $\odot$ is a mapping from D to R and satisfies the followings.

1. $\odot(a \otimes^{-1} b) = \odot a - \odot b$,
2. *if* $a < b$, *then* $\odot a < \odot b$,
3. $\odot \bot = \infty$.

In fact, if $\otimes^{-1}$ is $-$, $\odot$ can be $\equiv$; while if $\otimes^{-1}$ is $/$, $\odot$ can be ln.

Pl and Pl' are said to **have the same support** [CD05b] if $\forall w$, $Pl(w) \neq \bot$ iff $Pl'(w) \neq \bot$. If Pl and Pl' do not have the same support, as $\odot\bot = \infty$, we can conclude that $d(Pl, Pl') = \infty$.

Lemma 3 *For $a, b, c, d \in D$ and $a, b, c, d \neq \bot$, if $a \otimes^{-1} b \geq_D c \otimes^{-1} d$, we have $b \otimes^{-1} a \leq_D d \otimes^{-1} c$.*

Proposition 8 *$d(Pl, Pl')$ defined in Definition 7 is a distance function.*

A common perspective on revision strategies is to have minimal change between the prior belief (resp. epistemic state) and the revised belief (resp. epistemic state) [RF89], [KM91], [Bou96], [DP97]. The theorem below shows that our revision strategy is optimal in the sense that our revision strategy satisfies this common perspective.

Theorem 1 *The plausibility distribution Pl_1 obtained from Pl by plausibility kinematics on partition $A_1, A_2, \ldots, A_n$ of W is optimal in the following sense. Among all possible plausibility distributions that agree with Pl on the plausibility values of events $A_1, A_2, \ldots, A_n$, Pl_1 is the closest to Pl according to the distance measure by Definition 7.*

4 A verification using the belief revision postulates

In this section, we use some well known belief revision postulates to verify the revision operator by plausibility measures. We mainly adopt the postulates proposed by Darwiche and Pearl [DP97], and also consider the Recalcitrance postulate [NPP03] and the Independence postulate [JT07].

The Darwiche-Pearl iterated belief revision postulates (DP Postulates) [DP97], which stems from the KM postulates [KM91], provide a general framework as how a belief set shall be obtained after iterated belief revision. There are following postulates for general revision in which Φ stands for an epistemic state (usually it means W plus the preorder $\leq_\Phi$ on W) and $\Phi \circ \mu$ is a new epistemic state after revising Φ with revision operator $\circ$. For each epistemic state Φ, there is a belief set $Bel(\Phi)$ and it is defined as $Bel(\Phi) = \psi$, where $Mods(\psi) = min(W, \leq_\Phi)$. In the following when an epistemic state Φ is embedded in a logical formula, it actually represents its corresponding belief set. For example, $\Phi \wedge \mu$ stands for $Bel(\Phi) \wedge \mu$.

R1 $\Psi \circ \mu$ implies μ.
R2 If $\Psi \wedge \mu$ is satisfiable, then $\Psi \circ \mu \equiv \Psi \wedge \mu$.
R3 If μ is satisfiable, then $\Psi \circ \mu$ is also satisfiable.
R4 If $\Psi_1 = \Psi_2$ and $\mu_1 \equiv \mu_2$, then $\Psi_1 \circ \mu_1 \equiv \Psi_2 \circ \mu_2$.
R5 $(\Psi \circ \mu) \wedge \phi$ implies $\Psi \circ (\mu \wedge \phi)$.
R6 If $(\Psi \circ \mu) \wedge \phi$ is satisfiable, then $\Psi \circ (\mu \wedge \phi)$ implies $(\Psi \circ \mu) \wedge \phi$.

and the following postulates for iterated belief revision:

C1 If $\alpha \models \mu$, then $(\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha$.
C2 If $\alpha \models \neg\mu$, then $(\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha$.
C3 If $\Psi \circ \alpha \models \mu$, then $(\Psi \circ \mu) \circ \alpha \models \mu$.
C4 If $\Psi \circ \alpha \not\models \neg\mu$, then $(\Psi \circ \mu) \circ \alpha \not\models \neg\mu$.

The following two theorems are the representation theorems for the DP postulates.

Theorem 2 *([DP97]) A revision operator $\circ$ satisfies postulates R1-R6 precisely when the total pre-order $\leq_\Psi$ induced on the epistemic state Ψ satisfies:*

$$Mods(Bel(\Psi \circ \mu)) = min(Mods(\mu), \leq_\Psi), \textit{and}$$

1. $w_1, w_2 \models Bel(\Psi)$ *implies* $w_1 =_\Psi w_2$.
2. $w_1 \models Bel(\Psi)$ *and* $w_2 \models \neg Bel(\Psi)$ *implies* $w_1 \leq_{\Psi\circ\mu} w_2$.
3. $\Psi^1 = \Psi^2$ *implies* $\leq_{\Psi^1} = \leq_{\Psi^2}$.

Theorem 3 *([DP97]) Suppose that a revision operator $\circ$ satisfies postulates R1-R6. Then $\circ$ satisfies C1-C4 iff:*

CR1 *If $w_1 \models \mu$ and $w_2 \models \mu$, then $w_1 \leq_\Psi w_2$ iff $w_1 \leq_{\Psi\circ\mu} w_2$.*
CR2 *If $w_1 \models \neg\mu$ and $w_2 \models \neg\mu$, then $w_1 \leq_\Psi w_2$ iff $w_1 \leq_{\Psi\circ\mu} w_2$.*
CR3 *If $w_1 \models \mu$ and $w_2 \models \neg\mu$, then $w_1 <_\Psi w_2$ implies $w_1 <_{\Psi\circ\mu} w_2$.*
CR4 *If $w_1 \models \mu$ and $w_2 \models \neg\mu$, then $w_1 <_\Psi w_2$ iff $w_1 <_{\Psi\circ\mu} w_2$.*

We extend the plausibility measure Pl to propositions such that for a proposition μ, we have $Pl(\mu) = \oplus_{w \models \mu} Pl(w)$.

A proposition μ is believed by an agent if $Pl(\neg\mu) <_D \top$. An agent's belief in the current epistemic state Pl, denoted as $Bel(Pl)$, is then characterized as follows:

$$Mods(Bel(Pl)) \stackrel{def}{=} \{w : Pl(w) = \top\}.$$

Obviously, a proposition μ is accepted iff its models subsume $Mods(Bel(Pl))$, i.e., $Mods(Bel(Pl)) \subseteq Mods(\mu)$.

For a new piece of evidence, we assume that the evidence is represented as $Pl_e(\mu) = \top$ and $Pl_e(\neg\mu) < \top$. Furthermore, we assume $Pl_e(\neg\mu) = \beta \otimes Pl(\neg\mu)$ where $Pl(\neg\mu)$ is the plausibility measure of a prior belief and β is any value that satisfies $\beta \otimes Pl(\neg\mu) \neq \top$. Such β indeed exists, in fact, $\bot$ is such a value. Thus a revision operator $\bullet$ that revises Pl withe formula μ can be defined as

$$(Pl \bullet \mu)(w) \stackrel{def}{=} \begin{cases} Pl(w) \otimes^{-1} Pl(\mu) & \text{for } w \models \mu, \\ \beta \otimes Pl(w) & \text{for } w \models \neg\mu. \end{cases} \quad (6)$$

$Pl \bullet \mu$ is a new plausibility measure. In fact, $(Pl \bullet \mu)$ is equivalent to the (A, α)-revision $Pl_{A,\alpha}$ such that $A = Mods(\mu)$ and $\alpha = \beta \otimes Pl(\neg\mu)$.

Before discussing the relationship between the above revision operator $\bullet$ and the DP postulates, we introduce the following property.

Definition 8 *Let $\oplus$ be a mapping from $D \times D$ to D, then $\oplus$ is called Max-Additive iff it satisfies: $a \oplus b =_D a$ for $a, b \in D$ and $a \geq_D b$.*

For convenience, if Pl is associated with a Max-Additive $\oplus$, we simply call Pl is Max-Additive.

It is easy to find that if Pl is Max-Additive, then it is bounded-additive. And in fact it means that $\oplus$ is actually the *max* operator with respect to the total pre-order $\leq_D$. Obviously, OCFs and possibility measures satisfy this property, but probability functions do not. Intuitively this is not surprising, as OCFs and possibility measures have their belief sets whilst for probability functions, there are no corresponding belief sets.

Proposition 9 *Let $S = (W, \mathscr{F} = 2^W, D, Pl)$ be a plausibility space and the total pre-order on the set of interpretations is defined as*

$$w_1 \leq_{Pl} w_2 \stackrel{def}{=} Pl(w_1) \geq_D Pl(w_2).$$

Then we have:

1. $w_1, w_2 \models Bel(Pl)$ *implies* $w_1 =_{Pl} w_2$.
2. $w_1 \models Bel(Pl)$ *and* $w_2 \models \neg Bel(Pl)$ *implies* $w_1 \leq_{Pl \bullet \mu} w_2$.
3. $Pl^1 = Pl^2$ *implies* $\leq_{Pl^1} = \leq_{Pl^2}$.

and we also have

$$Mods(Bel(Pl \bullet \mu)) = min(Mods(\mu), \leq_{Pl})$$

iff Pl *is Max-Additive.*

Proposition 10 *Let* $\leq_{Pl}$ *and* $\leq_{Pl \bullet \mu}$ *be total pre-orders induced by* Pl *and* $Pl \bullet \mu$ *respectively, then we have:*

PlR1 *If* $w_1 \models \mu$ *and* $w_2 \models \mu$, *then* $w_1 \leq_{Pl} w_2$ *iff* $w_1 \leq_{Pl \bullet \mu} w_2$.
PlR2 *If* $w_1 \models \neg\mu$ *and* $w_2 \models \neg\mu$, *then* $w_1 \leq_{Pl} w_2$ *iff* $w_1 \leq_{Pl \bullet \mu} w_2$.
PlR3 *If* $w_1 \models \mu$ *and* $w_2 \models \neg\mu$, *then* $w_1 <_{Pl} w_2$ *implies* $w_1 <_{Pl \bullet \mu} w_2$.
PlR4 *If* $w_1 \models \mu$ *and* $w_2 \models \neg\mu$, *then* $w_1 <_{Pl} w_2$ *iff* $w_1 <_{Pl \bullet \mu} w_2$.

With Propositions 9 and 10, we immediately get that our revision operator satisfies all DP postulates (with the help of Theorems 2 and 3). Thus, for the Max-Additive plausibility measures, we have

Theorem 4 *The revision operator* $\bullet$ *defined in Equation 6 satisfies the DP postulates R1-R6 and C1-C4.*

The Recalcitrance (Rec) postulate [NPP03] and Independent (Ind) postulate [JT07] are presented as follows.

Rec If $\alpha \not\models \neg\mu$, then $(\Phi \circ \mu) \circ \alpha \models \mu$.
Ind If $\Phi \circ \neg\alpha \not\models \neg\mu$, then $(\Phi \circ \mu) \circ \neg\alpha \models \mu$.

Semantically, postulate Rec and Ind correspond to the following conditions ([NPP03] and [JT07]).

RecR If $w_1 \models \mu$ and $w_2 \models \neg\mu$, then $w_1 <_{\Phi \circ \mu} w_2$.
IndR If $w_1 \models \mu$ and $w_2 \models \neg\mu$, then $w_1 \leq_{\Phi} w_2$ only if $w_1 <_{\Phi \circ \mu} w_2$.

Thus, the following proposition shows that $\bullet$ operator defined by Equation 6 satisfies the Independence postulate.

Proposition 11 *Let* $\leq_{Pl}$ *and* $\leq_{Pl \bullet \mu}$ *be total pre-orders induced by* Pl *and* $Pl \bullet \mu$, *then we have:*

PlIndR *If* $w_1 \models \mu$ *and* $w_2 \models \neg\mu$, *then* $w_1 \leq_{Pl} w_2$ *only if* $w_1 <_{Pl \bullet \mu} w_2$.

hence the revision operator $\bullet$ *defined in Equation 6 satisfies the Independence Postulate.*

And the following example shows that the Recalcitrance postulate is not satisfied by $\bullet$.

Example 1 *Let* $W = \{w_1, w_2, w_3\}$, Pl *be an OCF* κ *(thus* $\otimes$ *is* $+$*) over* W *such that* $\kappa(w_1) = 3$, $\kappa(w_2) = 0$ *and* $\kappa(w_3) = 1$, *and* μ *be a formula such that* $Mods(\mu) = \{w_1, w_3\}$ *(thus* $\kappa(\mu) = 1$*), then let* $\beta = 1$, *we have* $(\kappa \bullet \mu)(w_1) = 2 > 1 = (\kappa \bullet \mu)(w_2)$ *which violates the RecR condition.*

5 Conclusion

In this paper, we presented a general revision model for epistemic state using plausibility measures and this model generalizes Spohn's and Dubois and Prade's results on revision in ordinal conditional functions and possibility theory. The reversible and commutative properties are proved to be held in our model. Moreover, we proposed a notion of plausibility kinematics which is a generalization of probability kinematics [Jef65] and showed that the revision using plausibility kinematics satisfies the principle of minimal change, so that our revision model to some extent is optimal. Finally, we used the DP postulates [DP97] to verify our revision operator and proved that our revision strategy and the DP postulates are compatible when plausibility measures satisfy the Max-Additive property.

In [Hal01], Halpern showed that variety of uncertainty measures can be represented by plausibility measures. Therefore, it would be interesting to see if our revision model can be applied to those uncertainty measures. Another issue for future research is that Darwiche and Pearl's iterated belief revision cannot be applied to probability measures, because there does not exist a belief set from a probability distribution. Therefore, more general revision postulates maybe required purely on epistemic states other than on their associated belief sets.

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