Modelling and Reasoning with Uncertain Event-Observations for Event Inference

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Abstract: This paper presents an event modelling and reasoning framework where event-observations obtained from heterogeneous sources may be uncertain or incomplete, while sensors may be unreliable or in conflict. To address these issues we apply Dempster-Shafer (DS) theory to correctly model the event-observations so that they can be combined in a consistent way. Unfortunately, existing frameworks do not specify which event-observations should be selected to combine. Our framework provides a rule-based approach to ensure combination occurs on event-observations from multiple sources corresponding to the same event of an individual subject. In addition, our framework provides an inference rule set to infer higher level inferred events by reasoning over the uncertain event-observations as epistemic states using a formal language. Finally, we illustrate the usefulness of the framework using a sensor-based surveillance scenario.

1 INTRODUCTION AND RELATED WORK

CCTV surveillance systems are deployed in various environments including airports (Weber and Stone, 1994), railways (Sun and Velastin, 2003), retail stores (Brodsky et al., 2001) and forensic applications (Geradts and Bijhold, 2000). Such systems detect, recognise and track objects of interest through gathering and analysing real-time event-observations from low-level sensors and video analytic components. This allows the system to take appropriate actions to stop or prevent undesirable behaviours e.g. petty crime or harassment. However, event-observations may be uncertain or incomplete (e.g. due to noisy measurements etc.) while the sensors themselves may be unreliable or in conflict (e.g. due to malfunctions, inherent design limitations). As such, an important challenge is how to accurately model and combine event-observations from multiple sources to ensure higher level inferred events that provide semantically meaningful information in an uncertain, dynamic environment.

In the literature, various event reasoning systems have been suggested for handling uncertainty in events (Wasserkrug et al., 2008; Ma et al., 2009). In particular, the framework proposed by (Wasserkrug et al., 2008) considers the uncertainty in event-observations and the uncertainty in rules. Specifically, this is modelled as a single Bayesian network which is continuously updated at run-time when new primitive event-observations are observed. Furthermore, inferred events are continuously recognised using probabilistic inference over the Bayesian network. In (Ma et al., 2009; Ma et al., 2010), the authors address the problem of uncertain and conflicting information from multiple sources. They use Dempster-Shafer (DS) theory of evidence (Shafer, 1976) to combine (uncertain) event-observations from multiple sources to find a representative model of the underlying sources. However, in (Ma et al., 2009; Ma et al., 2010) the authors do not specify what event-observations to combine. This is necessary to ensure combination occurs on event-observations from multiple sources corresponding to the same event of an individual subject. If this is not considered then the combined event-observation result will be inconsistent and not representative of the underlying sources. Furthermore, in (Ma et al., 2009; Ma et al., 2010), the authors use a rule-based inference system to derive inferred events from primitive event-observations. However, in (Ma et al., 2009; Ma et al., 2010) they do not define the formal semantics of the conditions within their inference rules.

The main contributions of this work are as follows:

(i) We revise and extend significantly the event modelling and reasoning framework of (Ma et al., 2009; Ma et al., 2010) using Dempster-Shafer theory.

(ii) We provide a rule-based approach to specify how to select the event-observations to be combined.

(iii) We use the framework of (Bauters et al., 2014) to reason over the uncertain information as probabilistic epistemic states using a formal language.

(iv) We present a scenario from a sensor-based surveillance system to illustrate our framework.

The remainder of this paper is organised as follows. In Section 2, we introduce the preliminaries of DS theory and modelling uncertain information as epistemic states. In Section 3, we propose our event modelling and reasoning framework. In Section 4, we present a sensor-based surveillance scenario to illustrate our framework. Finally, in Section 5 we conclude this paper and discuss future work.

2 PRELIMINARIES

In this section, we provide the preliminaries on Dempster-Shafer (DS) theory (Shafer, 1976) and how to model uncertain information as epistemic states.

2.1 Dempster-Shafer Theory

Dempster-Shafer theory is capable of dealing with incomplete and uncertain information.

Definition 1. *Let Ω be a set of exhaustive and mutually exclusive hypotheses, called a* frame of discernment. *A function* $\mathsf{m} : 2^{\Omega} \rightarrow [0,1]$ *is called a* mass *function over Ω if* $\mathsf{m}(\emptyset) = 0$ *and* $\sum_{A\subseteq\Omega} \mathsf{m}(A) = 1$. *Also, a belief function and plausibility function from* m, *denoted* Bel *and* Pl, *are defined for each $A \subseteq \Omega$ as:*

$$\mathsf{Bel}(A) = \sum_{B\subseteq A} \mathsf{m}(B),$$
$$\mathsf{Pl}(A) = \sum_{A\cap B\neq\emptyset} \mathsf{m}(B).$$

Any $A \subseteq \Omega$ such that $\mathsf{m}(A) > 0$ is called a *focal element* of m. Intuitively, $\mathsf{m}(A)$ is the proportion of evidence that supports A, but none of its strict subsets. Similarly, $\mathsf{Bel}(A)$ is the degree of evidence that the true hypothesis belongs to A and $\mathsf{Pl}(A)$ is the maximum degree of evidence supporting A. The values $\mathsf{Bel}(A)$ and $\mathsf{Pl}(A)$ represent the lower and upper bounds of belief, respectively.

To reflect the reliability of evidence we can apply a discounting factor to a mass function using Shafer's discounting technique (Shafer, 1976) as follows:

Definition 2. *Let* m *be a mass function over Ω and $\alpha \in [0,1]$ be a discount factor. Then a* discounted *mass function with respect to α, denoted* m^{α}, *is defined for each $A \subseteq \Omega$ as:*

$$\mathsf{m}^{\alpha}(A) = \begin{cases} (1-\alpha)\cdot\mathsf{m}(A), & \text{if } A \subset \Omega, \\ \alpha + (1-\alpha)\cdot\mathsf{m}(A), & \text{if } A = \Omega. \end{cases}$$

The effect of discounting is to remove mass assigned to focal elements and to then assign this mass to the frame. When α = 0, the source is completely reliable, and when α = 1, the source is completely unreliable. Once a mass function has been discounted, it is then treated as fully reliable.

One of the best known rules to combine mass functions is Dempster's rule of combination, which is defined as follows:

Definition 3. *Let* m_i *and* m_j *be mass functions over Ω from independent and reliable sources. Then the combined mass function using* Dempster's rule of combination, *denoted* $\mathsf{m}_i \oplus \mathsf{m}_j$, *is defined for each $A \subseteq \Omega$ as:*

$$(\mathsf{m}_i \oplus \mathsf{m}_j)(A) = \begin{cases} c\sum_{B\cap C=A} \mathsf{m}_i(B)\mathsf{m}_j(C), & \text{if } A \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

where $c = \frac{1}{1-\mathsf{K}(\mathsf{m}_i,\mathsf{m}_j)}$ *is a normalization constant with* $\mathsf{K}(\mathsf{m}_i,\mathsf{m}_j) = \sum_{B\cap C=\emptyset} m_i(B)m_j(C)$.

The effect of the normalization constant c, with $\mathsf{K}(\mathsf{m}_i,\mathsf{m}_j)$ the degree of conflict between m_i and m_j, is to redistribute the mass value assigned to the empty set.

To reflect the belief distributions from preconditions to the conclusion in an inference rule, in (Liu et al., 1992), a modelling and propagation approach was proposed based on the notion of evidential mapping Γ.

Definition 4. *Let* $\Gamma : \Omega_e \times 2^{\Omega_h} \rightarrow [0,1]$ *be an evidential mapping from frame Ω_e to frame Ω_h that satisfies the condition* $\Gamma(\omega_e,\emptyset) = 0$ *and* $\sum_{H\subseteq\Omega_h} \Gamma(\omega_e,H) = 1$. *Let Ω_e and Ω_h be frames, with* m_e *a mass function over Ω_e and Γ an evidential mapping from Ω_e to Ω_h. Then a mass function* m_h *over Ω_h is an evidence propagated mass function from* m_e *with respect to Γ and is defined for each $H \subseteq \Omega_h$ as:*

$$\mathsf{m}_h(H) = \sum_{E\subseteq\Omega_e} \mathsf{m}_e(E)\Gamma^*(E,H),$$

where: $\Gamma^*(E,H) =$

$$\begin{cases} i, & \text{if } H \neq \bigcup H_E \wedge \forall\omega_e \in E,\ \Gamma(\omega_e,H) > 0, \\ 1-j, & \text{if } H = \bigcup H_E \wedge \exists\omega_e \in E,\ \Gamma(\omega_e,H) = 0, \\ 1-i+j & \text{if } H = \bigcup H_E \wedge \forall\omega_e \in E, \Gamma(\omega_e,H) > 0, \\ 0, & \text{otherwise}, \end{cases}$$

with $i = \sum_{\omega_e \in E} \frac{\Gamma(\omega_e, H)}{|E|}$, $j = \sum_{H' \in H_E} \Gamma^*(E, H')$, $H_E = \{H' \subseteq \Omega_h \mid \omega_e \in E, \Gamma(\omega_e, H') > 0\}$ *and* $\bigcup H_E = \{\omega_h \in H' \mid H' \in H_E\}$.

2.2 Modelling uncertain information as epistemic states

To define an epistemic state we let At be a finite set of propositional atoms. Then for a set of atoms $A \subseteq At$, the set of literals given the atoms in A are: $lit(A) = \{a | a \in A\} \cup \{\neg a | a \in A\}$. A formula ϕ is defined in Backus-Naur Form (BNF) as $\phi ::= a | \neg a | (\phi_1 \wedge \phi_2) | (\phi_1 \vee \phi_2)$ where all formulas are in Negation Normal Form (NNF). Here, the language is denoted as $\mathcal{L}$. A function $\omega : At \to \{TRUE, FALSE\}$ is called a possible world (or interpretation) which assigns a truth value to every variable. The set of all possible worlds is denoted Ω. A possible world ω is a model of a formula ϕ iff the possible world ω makes ϕ true in the standard truth functional way, denoted as $\omega \models \phi$. The set of all models of ϕ is denoted as $mod(\phi)$. An epistemic state is defined as follows:

Definition 5. *(from (Ma and Liu, 2011)) Let* Ω *be a set of possible worlds. An epistemic state is a mapping* $\Phi : \Omega \to \mathbb{Z} \cup \{-\infty, +\infty\}$.

An epistemic state represents the state of the world where $\Phi(\omega)$ represents the degree of belief in a possible world ω. Then $\Phi(\omega) = +\infty$ indicates ω is fully plausible, $\Phi(\omega) = -\infty$ indicates ω is not plausible and $\Phi(\omega) = 0$ indicates total ignorance about ω. For $\omega, \omega' \in \Omega$ and $\Phi(\omega) > \Phi(\omega')$ then ω is more plausible than ω'.

To reason about epistemic states we consider the work of (Bauters et al., 2014). The language $\mathcal{L}$ is extended with the connectives $>$ and $\geq$ such that we have $\phi > \psi$ and $\phi \geq \psi$ respectively. The former means ϕ is strictly more plausible than ψ whereas the latter means ϕ is at least as plausible as ψ. The resulting language $\mathcal{L}^*$ can be defined in BNF as $\phi ::= a | \neg a | \phi_1 \wedge \phi_2 | \phi_1 \vee \phi_2 | \phi_1 > \phi_2 | \phi_1 \geq \phi_2$ where ϕ_1 is strictly more plausible than ϕ_2 if $\phi_1 > \phi_2$. Moreover, ϕ_1 is more plausible or equal to ϕ_2 if $\phi_1 \geq \phi_2$.

The semantics of the language $\mathcal{L}^*$ are defined using a mapping λ where formulas $\phi \in \mathcal{L}^*$ map onto $\mathbb{Z} \cup \{-\infty, +\infty\}$. Intuitively, $\lambda(\phi)$, associated with the formula ϕ reflects how plausible it is. However, if ϕ is not a propositional statement (i.e. $\phi \notin \mathcal{L}$) it becomes necessary to pare down the formula until it becomes a classical propositional statement. This is completed by the following definition:

Definition 6. *(from (Bauters et al., 2014)) Let* $\phi \in \mathcal{L}^*$ *be a formula in the extended language. Then when* $\phi \in \mathcal{L}$, $\lambda(\phi) = max\{\Phi(\omega) | \omega \models \phi\}$ *with* $max(\emptyset) = -\infty$. *Otherwise, we define* $\lambda(\phi) = \lambda(pare(\phi))$ *with pare defined as:*

$$pare(\phi \wedge \psi) = check(\phi) \wedge check(\psi)$$
$$pare(\phi \vee \psi) = check(\phi) \vee check(\psi)$$
$$pare(\phi \geq \psi) = \begin{cases} \top & if\, \lambda(\phi) \geq \lambda(\psi) \\ \bot & otherwise \end{cases}$$
$$pare(\phi > \psi) = \begin{cases} \top & if\, \lambda(\phi) > \lambda(\psi) \\ \bot & otherwise \end{cases}$$
$$pare(not \phi) = \begin{cases} \top & if\, \phi \in \mathcal{L}\ and\ \lambda(\neg\phi) \geq \lambda(\phi) \\ \bot & otherwise \end{cases}$$
$$check(\phi) = \begin{cases} \phi & if\, \phi \in \mathcal{L} \\ pare(\phi) & otherwise \end{cases}$$

with $\top$ *a tautology (i.e. true) and* $\bot$ *an inconsistency (i.e. false).*

The intuition of paring down is straightforward: for the operators $\wedge$ and $\vee$ we verify if it is a formula in the language $\mathcal{L}$. Otherwise, we need to pare it down to get a propositional formula. When the operator is either $>$ or $\geq$, we define it as $\phi > \psi$ which is read as 'ϕ is more plausible than ψ' or 'we have less reason to believe $\neg\phi$ than $\neg\psi$'. This will always evaluate to true or false, i.e. $\top$ or $\bot$.

Using the λ-mapping we now define when a formula ϕ is entailed.

Definition 7. *(from (Bauters et al., 2014)) Let* Φ *be an epistemic state and* ϕ *a formula in* $\mathcal{L}^*$. *We say that* ϕ *is entailed by* Φ, *written as* $\Phi \models \phi$, *if and only if* $\lambda(\phi) > \lambda(\neg\phi)$.

3 REASONING ABOUT UNCERTAIN EVENT-OBSERVATIONS

In this section we propose a new event modelling and reasoning framework by revising and extending the framework of (Ma et al., 2009; Ma et al., 2010). Initially, we formally define an event model to represent the attributes and semantics of event-observations detected from information provided by various sources. This ensures that the event-observations themselves are represented and reasoned as well as allowing inferences to be made subsequently. Events can be classified as (i) external events which are those directly gathered from external sources or (ii) inferred events which are the result of the inference rules of the event model.

3.1 Event Detection

Let Υ be a non-empty finite set of variables. The frame (or set of possible values) associated with a variable $\upsilon \in \Upsilon$ is denoted Ω_υ. For a set of variables $\Psi \subseteq \Upsilon$, the (product) frame Ω_Ψ is defined as $\prod_{\upsilon\in\Psi} \Omega_\upsilon$.

Definition 8. *Let $[t,t']$ be an interval of time starting at timepoint t and ending at timepoint t', s be a source, p be a subject, Ψ be a set of variables and* m *be a mass function over Ω_Ψ. Then a tuple $e = ([t,t'], s, p, \mathsf{m})$ is called an event-observation.*

The mass function m represents some (uncertain) event-observation, made by source s at its temporal location $[t,t']$, for some real-world event. We assume that a source can only make one event-observation at timepoint t about an individual subject p[1]. Furthermore, a source can detect multiple event-observations at any one time as long as they correspond to different subjects.

Definition 9. *Let S be a set of sources. A function $r : S \mapsto [0,1]$ is called a source reliability measure such that $r(s) = 1$ if s is completely reliable, $r(s) = 0$ if s is completely unreliable and $r(s) \geq r(s')$ if s is at least as reliable as (s').*

Information related to event-observations are modelled as mass functions. However, due to their reliability we apply their source reliability measure to derive discounted mass functions that can then be treated as fully reliable. The reliability measure of each source is based on its historical data, age, design limitations and faults etc. A primitive event set $\mathsf{E_P}$ will contain a set of event-observations $e_1 \dots e_n$ with their corresponding discounted mass functions $\mathsf{m}_1^\alpha \dots \mathsf{m}_n^\alpha$.

Example 1. Consider two independent sources s_1 and s_2 where s_1 is CCTV which detects the behaviour of a subject named Alice ($\Omega_{\Psi_b} = \{walking, sitting\}$) and is 90% reliable and s_2 is a thermometer which detects the temperature of a room ($\Omega_{\Psi_t} = \{0,\dots,50\}$) and is 85% reliable. Information has been obtained from 9 am such that s_1:[*walking*(80% certain)] and $s_2[30^oC]$. By modelling the (uncertain) information as mass functions we have $\mathsf{m}_1(\{walking\}) = 0.8$, $\mathsf{m}_1(\Omega_{\Psi_b}) = 0.2$ and $\mathsf{m}_2(\{30\}) = 1$, respectively. Given this information we have the following event-observations in $\mathsf{E_P}$:

$$e_1 = ([900,901], s_1, Alice, \mathsf{m}_1^{0.1}(\{walking\}) = 0.72, \mathsf{m}_1^{0.1}(\Omega_{\Psi_b}) = 0.28),$$
$$e_2 = ([900,902], s_2, -, \mathsf{m}_2^{0.15}(\{30\}) = 0.85, \mathsf{m}_2^{0.15}(\Omega_{\Psi t}) = 0.15).$$

For these event-observations we apply the discount factors (i.e. α = 0.1 and 0.15 respectively) for s_1 and s_2 to the mass functions m_1 and m_2 to obtain the discounted mass functions.

Definition 10. *An event model M is defined as a tuple $\langle \mathsf{E_P}, \mathsf{E_C}, \mathsf{E_I} \rangle$ where $\mathsf{E_P}$ is a primitive event set (a set of discounted event-observations), $\mathsf{E_C}$ is a combined event set (a set of combined event-observations) and $\mathsf{E_I}$ is an inference event set (a set of inferred events).*

3.2 Event-Observation Combination Constraints

In some situations, we need to combine mass functions which relate to different attributes. However, combination rules require that mass functions have the same frame. The vacuous extension is a tool used for defining mass functions on a compatible frame.

A mass function $\mathsf{m}^{\Omega_{\Psi_1}}$, expressing a state of belief on Ω_{Ψ_1} is manipulated to a finer frame Ω_{Ψ_n}, a refinement of Ω_{Ψ_1} using the vacuous extension operation (Wilson, 2000), denoted by $\uparrow$. The vacuous extension of $\mathsf{m}^{\Omega_{\Psi_1}}$ from Ω_{Ψ_1} to the product frame $\Omega_{\Psi_1} \times \Omega_{\Psi_n}$, is obtained by transferring each mass $\mathsf{m}^{\Omega_{\Psi_1}}$, for any subset A of Ω_{Ψ_1} to the cylindrical extension of A.

Definition 11. *Let Ψ_1, Ψ_n be sets of variables and $\mathsf{m}_1, \dots, \mathsf{m}_n$ be mass functions over frames $\Omega_{\Psi_1}, \Omega_{\Psi_n}$. Then by vacuous extension, we obtain mass functions over $\Omega_{\Psi_1 \times \cdots \times \Psi_n}$, denoted $\mathsf{m}^{\Omega_{\Psi_1} \uparrow (\Omega_{\Psi_1} \times \cdots \times \Omega_{\Psi_n})}$ where*

$$\mathsf{m}^{\Omega_{\Psi_1} \uparrow (\Omega_{\Psi_1} \times \cdots \times \Omega_{\Psi_n})}(B) = \begin{cases} \mathsf{m}^{\Omega_{\Psi_1}}(A), & \text{if } B = A \times \Omega_{\Psi_2} \times \cdots \times \Omega_{\Psi_n}, \\ & A \subseteq \Omega_{\Psi_1} \\ 0, & \text{otherwise.} \end{cases}$$

Here, the propagation brings no new evidence and the mass functions are strictly equivalent (in terms of information).

Event-observations detected from various sources relating to a specific feature or subject identification will be combined at time T. The original and most common method of combining mass functions is using Dempster's combination rule. However, other combination operators such as the context-dependent combination rule from (Calderwood et al., 2016) or the disjunctive combination rule from (Dubois and

[1]We use " – " to denote an event-observation that does not have a subject.

Prade, 1992) may be more suitable given the information obtained from the sources.

Definition 12. *Let* $\{e_1,\ldots,e_n\}$ *be a set of observations from* $\mathsf{E_P}$ *and* $\{t,\ldots,t'\}$ *be a set of timepoints in* T *from* $e_1,\ldots,e_n$. *Then the combined observation* $e_1 \circ \cdots \circ e_n$ *is defined as:*

$$([\min(T_{min}),\max(T_{max})], s_1 \wedge \cdots \wedge s_n, \mathsf{m}_1^{'} \oplus \cdots \oplus \mathsf{m}_n^{'}),$$

where $[\min(T_{min}),\max(T_{max})]$ *represents the minimum and maximum of the set of timepoints,* $\mathsf{m}_i^{'} = \mathsf{m}_i^{\uparrow \Psi_1 \times \cdots \times \Psi_n}$ *and* $\mathsf{m}_i^{'} = \mathsf{m}_i^{1-r(s_1)}$. *Given that each* $\mathsf{m}_i^{'}$ *is a discounted mass function, we have that* $r(s_1 \wedge \cdots \wedge s_n) = 1$.

Notably, we can combine event-observations when their time interval overlaps. When this occurs, we select the earliest (resp. latest) timepoint from the events to use as timepoint t (resp. t') of the combined event-observation. For example, assume event-observations e_1 and e_2 are detected at $[900,905]$ and $[902,907]$ respectively. Then, the interval of time for a combined event-observation is $[900,907]$.

However, before a combination occurs we need to decide which event-observations are selected to combine.

Definition 13. *An event-observation constraint rule* c *is defined as a tuple*

$$\mathsf{c} = (TS, S, p)$$

where TS *is a time span constraint,* S *is a set of sources whose mass functions are to be combined from event-observations in* $\mathsf{E_P}$ *and* p *is a subject that is assigned to event-observations.*

In this work, we consider time, source and subject constraints. We combine event-observations obtained from multiple sources when they relate to the same event of an individual subject and are within a reasonable time span[2]. For example, event-observations relating to a subject p from sources s_1 and s_2 will be combined if they have been obtained at the same time.

Example 2. Consider the following constraint rules in the event-observation constraint rule set:

$$\mathsf{c}_1 = (5, \{s_1, s_3\}, p);$$
$$\mathsf{c}_2 = (0, \{s_2, s_4, s_5, s_6\}, -).$$

Assume we have six independent sources $s_1,\ldots,s_6$ where

(i) event-observations from sources s_1 and s_3 are obtained (at 9:00:35 am and 9:00:40 am respectively) about a subject named Alice;

(ii) event-observations from s_2, s_4, s_5 and s_6 are obtained (at 9 am) about the thermometers observing themselves.

Given the event-observations from (i), the constraint rule c_1 is selected to combine the event-observations from sources s_1 and s_3 as the timespan is within 5 seconds and they relate to a subject named Alice. Furthermore, given the event-observations from (ii), the constraint rule c_2 is selected to combine the event-observations from sources s_2, s_4, s_5 and s_6 as they were all obtained at 9 am and they relate to the same type of source i.e. thermometers.

The semantics for combining event-observations is as follows:

Definition 14. *Let* c *be an event-observation constraint rule,* $\langle \mathsf{E_P}, \mathsf{E_C}, \mathsf{E_I} \rangle$ *be the event model and* $E \subseteq \mathsf{E_P}$ *be a set of event-observations. Then the event-observation combination with respect to* c *be defined as:*

$$\frac{E \wedge \mathsf{c} \not\models \perp \ s.t. \forall E' \subset E, E' \wedge \mathsf{c} \models \perp}{\langle \mathsf{E_P}, \mathsf{E_C}, \mathsf{E_I} \rangle \rightarrow \langle \mathsf{E_P} \backslash E, \mathsf{E_C} \cup \{e_1 \circ \cdots \circ e_n\}, \mathsf{E_I} \rangle} \textit{combine}.$$

3.3 Event Inference

Event inferences are expressed as a set of inference rules which are used to represent the relationship between primitive and combined event-observations. New inferred events are derived which are more meaningful and highly significant. For example, a person entering a building at night may imply its either a staff member or an unauthorised person. However if further event-observations are obtained and combined where the person is a male with their face obscured then it may imply a higher level threat such as a theft.

In our framework, an epistemic state is instantiated to a mass function as follows:

Definition 15. *(adapted from Definition 5) Let* Ω *be a set of possible worlds. An epistemic state is a mapping* $\Phi : 2^{\Omega} \rightarrow \mathbb{Z} \cup \{-\infty, +\infty\}$.

In this paper, we define an inference rule as follows:

Definition 16. *An inference rule* i *is defined as a tuple*

$$\mathsf{i} = (TS, \phi, \Gamma_{\Psi}^{\Phi})$$

where TS *is the time span,* $\phi \in \mathcal{L}^*$ *is a formula and* Γ *is a multi-valued mapping that propagates a mass function from an epistemic state in* Ω_{Φ} *to a new mass function in* Ω_{Ψ}.

Notably, Ω_{Υ} and Ω_{Ψ} are some product frames for sets of variables Υ and Ψ, respectively (as in the first

[2]This will be domain-specific.

paragraph of Section 3.1). Moreover, $\mathcal{L}^*$ is the language defined over the same set of attributes (atoms) where a condition is always a formula in $\mathcal{L}^*$. Here, a formula can be equivalent to a possible world when there is a conjunction of literals. Thus, an inference rule defines a condition over some set of attributes and then uses evidential mapping to propagate the mass function from the possible worlds Φ of an epistemic state to the new mass function (that is related to a different set of attributes). The latter will be included in the new inferred event.

We will now explore the inference rule in more detail.

A: Formula - ϕ

Initially, we have a set of possible worlds Ω and a mass function over Ω (which is domain-specific). Relevant mass functions (from discounting or vacuous extension (see Definition 11) are then combined using Dempster's rule to provide a combined mass function.

Example 3. Let A_1 and A_2 be independent attributes where $\Omega_1 = \{a,b\}$ and $\Omega_2 = \{p,q\}$ are their possible values, respectively. Let m_1 and m_2 be mass functions over Ω such that:

$$\mathsf{m}_1(\{a\}) = 0.72, \mathsf{m}_1(\Omega_1) = 0.28,$$
$$\mathsf{m}_2(\{p\}) = 0.765, \mathsf{m}_2(\Omega_2) = 0.235.$$

By vacuously extending Ω_1 with Ω_2 we obtain $\Omega_1 \times \Omega_2 = \{(a,p),(a,q),(b,p),(b,q)\}$. Then by vacuously extending m_1 and m_2 to mass functions over $\Omega_1 \times \Omega_2$, we have:

$$\mathsf{m}_1(\{(a,p),(a,q)\}) = 0.72, \mathsf{m}_1(\Omega_1 \times \Omega_2) = 0.28,$$
$$\mathsf{m}_2(\{(a,p),(b,p)\}) = 0.765, \mathsf{m}_2(\Omega_1 \times \Omega_2) = 0.235.$$

By combining the new mass functions m_1 and m_2 using Dempster's rule we have a new combined mass function m as follows:

$$\mathsf{m}(\{(b,p),(a,p)\}) = 0.214, \mathsf{m}(\{(a,p)\}) = 0.551,$$
$$\mathsf{m}(\{(a,q),(a,p)\}) = 0.169, \mathsf{m}(\Omega_1 \times \Omega_2) = 0.066.$$

To evaluate the formulas we use the plausibility function from DS theory. The justification being that possibilistic logic uses the possibility measure, and the possibility measure in possibility theory is comparable to the plausibility function in DS theory. As such, we instantiate Definition 6 and Definition 7 as follows: $\lambda(\phi) = \mathsf{Pl}(\phi) = \mathsf{Pl}(mod(\phi))$ where the λ-mapping has been instantiated with the plausibility function. This means, by definition, we have that $\mathsf{Pl}(\top) = 1$ and $\mathsf{Pl}(\bot) = 0$.

Example 4. (Continuing Example 3) Let $A_1 = a$, $A_2 = p$ and $A_1 = a \wedge A_2 = p$ be formulas[3]. Then

(i) $mod(A_1 = a) = \{(a,p),(a,q)\}$,
(ii) $mod(A_2 = p) = \{(a,p),(b,p)\}$,
(iii) $mod(A_1 = a \wedge A_2 = p) = \{(a,p)\}$,
(iv) $mod(\neg(A_1 = a \wedge A_2 = p)) = \{(a,q),(b,p)(b,q)\}$.

The plausibility values for the formulas are:

(i) $\mathsf{Pl}(A_1 = a)$ =1,
(ii) $\mathsf{Pl}(A_2 = p)$ =1,
(iii) $\mathsf{Pl}(A_1 = a \wedge A_2 = p) = 1$,
(iv) $\mathsf{Pl}(\neg(A_1 = a \wedge A_2 = p)) = 1$.

Then $\mathsf{Pl}(A_1 = a \wedge A_2 = p) \not> \mathsf{Pl}(\neg(A_1 = a \wedge A_2 = p))$. The formula $A_1 = a \wedge A_2 = p$ is not entailed.

B: Evidential mapping - Γ_Ψ^Φ

Evidential mappings are defined from a mass function in an epistemic state Ω_Φ to a new mass function in Ω_Ψ. These mappings allow us to derive a mass function to be included as part of the new inferred event.

Example 5. Consider Table 1 where $\Omega_\Psi = \{c_1,c_2,c_3\}$ represents conclusion 1, conclusion 2 and conclusion 3, respectively.

Table 1: Evidential mapping Γ_Ψ^Φ from $\Omega_\Phi = \{(a,p),(a,q),(b,p),(b,q)\}$ to $\Omega_\Psi = \{c_1,c_2,c_3\}$.

	$\{c_1\}$	$\{c_1,c_2\}$	$\{c_2,c_3\}$	$\{c_3\}$
(a,p)	0.5	0.5	0	0
(a,q)	0	0	1	0
(b,p)	0	0	0.7	0.3
(b,q)	0	0	0.5	0.5

From the evidential mapping in Table 1 and the combined mass function (from Example 3), then a mass function m_Ψ over Ψ is the evidence propagated mass function from m_Φ with respect to Γ such that:

$$\mathsf{m}_\Psi(\{c3\}) = 0.045, \mathsf{m}_\Psi(\{c1,c2\}) = 0.38,$$
$$\mathsf{m}_\Psi(\{c1\}) = 0.38, \mathsf{m}_\Psi(\{c2,c3\}) = 0.196.$$

Example 6. Let i_1 be an inference rule. Then we have:

$$\mathsf{i}_1 = (0,(A_1 = a \wedge A_2 = p),\Gamma_\Psi^\Phi).$$

where Γ_Ψ^Φ is an evidential mapping as shown in Table 1.

The semantics of event inference is defined as follows:

Definition 17. *Let* i *be an inference rule,* ϕ *be the formula from* i, $\langle \mathsf{E_P},\mathsf{E_C},\mathsf{E_I}\rangle$ *be the event model,* $E \subseteq \mathsf{E_P} \cup \mathsf{E_C} \cup \mathsf{E_I}$ *be a set of event-observations from the primitive event set, combined event set and inference event set respectively. Then the inference rule selection with respect to* i *be defined as:*

$$\frac{E \models \phi}{\langle \mathsf{E_P},\mathsf{E_C},\mathsf{E_I}\rangle \rightarrow \langle \mathsf{E_P},\mathsf{E_C},\mathsf{E_I} \cup \{e_1^I,\ldots,e_n^I\rangle} infer.$$

[3] Assume ordering (A_1,A_2).

Notably, an inferred event e_i^I will be defined similar to that of a primitive event-observation (see Definition 8) except that in this case its mass function is an evidence propagated mass function and its source will be a set of sources.

Now, we can extend the definition of the event model M as follows:

Definition 18. *An event model M^* is defined as a tuple $\langle \mathsf{E_P}, \mathsf{E_C}, \mathsf{E_I}, \mathsf{C}, \mathsf{I} \rangle$ where C is an event-observation combination constraints rule set, I is an inference rule set and the other items are the same as those defined in Definition 10.*

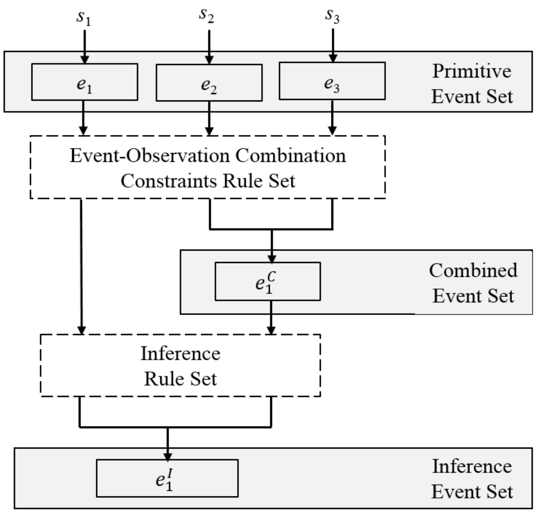

Figure 1: An illustration of the event model M^* using primitive and combined event-observations to infer an inferred event.

4 Surveillance System Scenario

In this section, we consider a scenario from a surveillance system to illustrate our event modelling and reasoning framework. Specifically, we monitor a single subject[4] i.e. a staff member enter a computer room at 10 am by swiping their access card (see Figure 2 (i)). Information is retrieved for that staff member and they are assigned an id e.g. $p_1, \ldots, p_n$. Within the room, the staff member will fulfil their job therefore their behaviour is expected to coincide with their job role. For example, a technician may enter the room, walk towards a computer, sit down at a computer and access the network. However, a cleaner may enter the room, walk towards a computer and dust the desk. A violation will occur if the behaviour of that staff member does not match that expected of their job. For example, given Figure 2 (ii) it shows a staff member sitting at the computer. This behaviour is normal for a technician but a violation for a cleaner.

Figure 2: A staff member (i) entering a computer room and (ii) sitting at a computer.

4.1 Event Detection

4.1.1 Primitive Event-Observations

In the surveillance system, a number of heterogeneous sources with various levels of granularity (e.g. cameras, microphones) will identify and monitor the behaviour of each staff member through classification analysis etc. Sources s_1, s_2 and s_3 are cameras located within the computer room where source s_1 detects the obfuscation of a staff member i.e. $\Omega_o = \{obscured, \neg obscured\}$ and is 90% reliable, s_2 detects the gender of a staff member i.e. $\Omega_g = \{male, female\}$ and is 70% reliable and s_3 detects the behaviour of a staff member i.e. $\Omega_b = \{walking, sitting\}$[5] and is 90% reliable. Furthermore, sources s_4 and s_5 are light sensors which detect light in the computer room i.e. $\Omega_l = \{on, off\}$. These sources are 60% and 90% reliable, respectively. In this scenario, we assume the event-observations from sources s_1, s_2 and s_3 were obtained from 10 am for a subject p_1 and from sources s_4 and s_5 for the light sensor. We have:

$$\begin{aligned} s_1 &: [obscured(80\%certain)], \\ s_2 &: [male(70\%certain)], \\ s_3 &: [walking(80\%certain)], \\ s_4 &: [on(70\%certain)], \\ s_5 &: [on(90\%certain)]. \end{aligned}$$

By modelling the (uncertain) information as mass

[4]For multiple subjects, algorithms for video classification or tracking etc. will detect each individual subject and assign an unique id.

[5]Notably, the behaviour of a staff member can be extended to the following: $\Omega_b = \{walking, sitting, running, standing, loitering, \ldots\}$

functions we have:

$$\mathsf{m}_1(\{obscured\}) = 0.8, \mathsf{m}_1(\Omega_{\Psi_o}) = 0.2,$$
$$\mathsf{m}_2(\{male\}) = 0.7, \mathsf{m}_2(\Omega_{\Psi_g}) = 0.3,$$
$$\mathsf{m}_3(\{walking\}) = 0.8, \mathsf{m}_3(\Omega_{\Psi_b}) = 0.2,$$
$$\mathsf{m}_4(\{on\}) = 0.7, \mathsf{m}_4(\Omega_{\Psi_l}) = 0.3,$$
$$\mathsf{m}_5(\{on\}) = 0.9, \mathsf{m}_5(\Omega_{\Psi_l}) = 0.1.$$

Given this information we have the following primitive event-observations in E_P:

$$e_1 = ([1000, 1001], s_1, p_1, \mathsf{m}_1^{0.1}(\{obscured\}) = 0.72, \mathsf{m}_1^{0.1}(\Omega_{\Psi_o}) = 0.28)),$$
$$e_2 = ([1000, 1001], s_2, p_1, \mathsf{m}_2^{0.3}(\{male\}) = 0.49, \mathsf{m}_2^{0.3}(\Omega_{\Psi g}) = 0.51)),$$
$$e_3 = ([1001, 1002], s_3, p_1, \mathsf{m}_3^{0.1}(\{walking\}) = 0.72, \mathsf{m}_3^{0.1}(\Omega_{\Psi b}) = 0.28)),$$
$$e_4 = ([1000, 1015], s_4, -, \mathsf{m}_4^{0.4}(\{on\}) = 0.42, \mathsf{m}_4^{0.4}(\Omega_{\Psi l}) = 0.58)),$$
$$e_5 = ([1000, 1015], s_5, -, \mathsf{m}_5^{0.1}(\{on\}) = 0.81, \mathsf{m}_5^{0.1}(\Omega_{\Psi l}) = 0.19)).$$

Notably, we have applied the discount factors (i.e. α = 0.1, 0.3, 0.1, 0.4 and 0.1 respectively) for sources $s_1, ... s_5$ to obtain the discounted mass functions for the event-observations $e_1, ..., e_5$ respectively.

In a real world surveillance system, event-observations will be continuously detected about each subject over a period of time. As such, further event-observations may include the following for the subject p_1:

$$e_6 = ([1002, 1015], s_3, p_1, \mathsf{m}_3^{0.1}(\{sitting\}) = 0.72, \mathsf{m}_3^{0.1}(\Omega_{\Psi b}) = 0.28)),$$
$$e_7 = ([1002, 1002], s_7, p_1, \mathsf{m}_6^{0.1}(\{wakeup\}) = 0.72, \mathsf{m}_6^{0.1}(\Omega_{\Psi n}) = 0.28)),$$
$$e_8 = ([1003, 1015], s_7, p_1, \mathsf{m}_7^{0.1}(\{login\}) = 0.72, \mathsf{m}_7^{0.1}(\Omega_{\Psi n}) = 0.28)),$$
$$\cdots$$

Here, the event-observation e_6 shows that source s_3 detects the subject p_1 sitting. Furthermore, the event-observations e_7 and e_8 relate to the network events i.e. $\Omega_n = \{wakeup, sleep, login, logoff\}$ detected by s_7 which is 90% reliable. In both of these event-observations the information obtained was 80% certain. It is also worth noting that further event-observations can be obtained to account for the real complexity in a working surveillance system e.g. considering other sensor information on multiple subjects (obtained from multiple sources) and from various measurement devices.

4.1.2 Event-Observation Combination Constraints Rule Set

Consider the following rules:

$$\mathsf{c}_1 = (60, \{s_1, s_2\}, p_1),$$
$$\mathsf{c}_2 = (0, \{s_4, s_5\}, -).$$

The rule c_1 states that event-observations from sources s_1 and s_2 will be combined if there time span is within 60 seconds and they correspond to the same event for a staff member p_1. The rule c_2 states that event-observations from sources s_4 and s_5 will be combined if they have been obtained at the same time. Notably, in this scenario s_3 is not combined with other sources therefore we do not need a rule.[6]

4.1.3 Combined Event-Observations

In the surveillance system, it becomes necessary to define mass functions onto the same frame. By vacuously extending Ω_{Ψ_o} with Ω_{Ψ_g} we obtain

$$\Omega_{\Psi_o, \Psi_g} = \Omega_{\Psi_o} \times \Omega_{\Psi_g} = \{(obscured, male), (obscured, female), (\neg obscured, male), (\neg obscured, female)\}.$$

By using the constraint rule c_1 and Dempster's combination rule we obtain $\mathsf{m}_1 \oplus \mathsf{m}_2$ for subject p_1, resulting in the following combined observation m_1^C in E_C:

$$e_1^C = ([1000, 1001], \{s_1, s_2\}, p_1,$$
$$\mathsf{m}_1^C(\{(\neg obscured, male), (obscured, male)\}) = 0.137,$$
$$\mathsf{m}_1^C(\{(obscured, male)\}) = 0.353,$$
$$\mathsf{m}_1^C(\{(obscured, female), (obscured, male)\}) = 0.367,$$
$$\mathsf{m}_1^C(\Omega_o \times \Omega_p) = 0.143.$$

Notably, sources s_4 and s_5 will not be combined with sources s_1 and s_2 as s_4 and s_5 are from different sources, they do not correspond to the same event and they are not associated with subject p_1.

By using the constraint rule c_2 and Dempster's combination rule we obtain $\mathsf{m}_4 \oplus \mathsf{m}_5$ for the thermometer readings, resulting in the following combined observation m_2^C in E_C:

$$e_2^C = ([1000, 1015], \{s_4, s_5\}, -,$$
$$\mathsf{m}_2^C(\{on\}) = 0.89, \mathsf{m}_2^C(\Omega_l) = 0.11).$$

[6] In the real world, s_3 will be combined with multiple sources to detect the behaviour of a subject.

4.2 Event Inference

4.2.1 Inference Rules

In the surveillance system we have inference rules such as the following:

$$\mathsf{i}_1 = (0, (obfuscation = obscured \wedge gender = male \wedge behaviour = walking), \Gamma_{\Psi}^{\Phi}),$$

$$\mathsf{i}_2 = (0, (obfuscation = obscured \wedge gender = male \wedge behaviour = sitting), \Gamma_{\Psi}^{\Phi}),$$

$$\mathsf{i}_3 = (0, (light = on), \Gamma_{\Psi_2}^{\Phi_1}),$$

where Γ from $\Omega_\Phi = \{(obscured, male, walking), \ldots, (\neg obscured, female, sitting)\}$ to $\Omega_\Psi = \{l, m, h\}$ which represents the threat classifications of low level, moderate level and high level, respectively.

The rule i_1 states that if an obscured male is currently walking then it infers a moderate-high level threat (or an obscured male walking). The next rule i_2 states that if an obscured male is sitting at the computer then it infers a high level threat (or an obscured male sitting at a computer). Notably, further rules can be added to this rule set to infer further events of interest. For example, given the event-observations e_6, e_7 and e_8 we could have a rule to state that an obscured male is sitting at a computer and has logged on to the network.

Let *obfuscation*, *gender*, *behaviour* be attributes, (denoted as O, G and B, respectively) where $\Omega_o = \{obscured, \neg obscured\}$, $\Omega_g = \{male, female\}$ and $\Omega_b = \{walking, sitting\}$ are their possible values (denoted as $\Omega_o = \{o, \neg o\}$, $\Omega_g = \{m, f\}$ and $\Omega_b = \{w, s\}$, respectively). Let $O = o$, $G = m$, $B = w$ and $O = o \wedge G = m \wedge B = w$ be formulas[7]. Then:

(i) $mod(O = o) = \{(o,m,w), (o,m,s), (o,f,w), (o,f,s)\}$,

(ii) $mod(G = m) = \{(o,m,w), (o,m,s), (\neg o,m,w), (\neg o,m,s)\}$,

(iii) $mod(B = w) = \{(o,m,w), (o,f,w), (\neg o,m,w), (\neg o,f,w)\}$,

(iv) $mod(O = o \wedge G = m \wedge B = w) = \{(o,m,w)\}$,

(v) $mod(\neg(O = o \wedge G = m \wedge B = w)) = \{(o,m,s), \ldots, (\neg o,f,w)\}$.

The plausibility values for the formulas are:

(i) $\mathsf{Pl}(O = o) = 1$,

(ii) $\mathsf{Pl}(G = m) = 1$,

(iii) $\mathsf{Pl}(B = w) = 1$,

(iv) $\mathsf{Pl}(O = o \wedge G = m \wedge B = w) = 1$,

(v) $\mathsf{Pl}(\neg(O = o \wedge G = m \wedge B = w)) = 1$.

Then $\mathsf{Pl}(O = o \wedge G = m \wedge B = w) \not> \mathsf{Pl}(\neg(O = o \wedge G = m \wedge B = w))$. The formula $O = o \wedge G = m \wedge B = w$ is not entailed.

Alternatively, consider the light attribute (denoted as L) where $\Omega_l = \{on, off\}$. Let $L = on$ be a formula. Then the set of models are:

(i) $mod(L = on) = \{(on)\}$,

(ii) $mod(\neg(L = on)) = \{(off)\}$.

The plausibility values for the formulas are:

(i) $\mathsf{Pl}(L = on) = 1$,

(ii) $\mathsf{Pl}(\neg(L = on)) = 0.11$.

Then $\mathsf{Pl}(L = on) > \mathsf{Pl}(\neg(L = on))$ as $1 > 0.11$. The formula $L = on$ is entailed.

Table 2: Evidential mapping from $\Omega_\Phi = \{(o,m,w), \ldots, (\neg o, f, s)\}$ to $\Omega_\Psi = \{l, m, h\}$.

	$\{l\}$	$\{l,m\}$	$\{m\}$	$\{m,h\}$	$\{h\}$
(o,m,w)	0	0	0.25	0.75	0
(o,m,s)	0	0	0	0.6	0.4
(o,f,w)	0	0	0.25	0.75	0
(o,f,s)	0	0	0	0.6	0.4
$(\neg o,m,w)$	1	0	0	0	0
$(\neg o,m,s)$	0.9	0.1	0	0	0
$(\neg o,f,w)$	1	0	0	0	0
$(\neg o,f,s)$	0.9	0.1	0	0	0

Given the evidential mapping from Table 2 and the combined mass function, then a mass function m_Ψ over Ψ is the evidence propagated mass function from m_Φ with respect to Γ such that:

$$\mathsf{m}_\Psi(\{m,h\}) = 0.648, \mathsf{m}_\Psi(\{h\}) = 0.054,$$
$$\mathsf{m}_\Psi(\{l,m\}) = 0.003, \mathsf{m}_\Psi(\{l\}) = 0.167,$$
$$\mathsf{m}_\Psi(\{m\}) = 0.189.$$

Given the event-observations obtained from 10 am, we have the following inferred event in E_I:

$$e_1^I = ([1000, 1015], \{s_1, s_2, s_3\}, p_1,$$
$$\mathsf{m}_1^I(\{m,h\}) = 0.648, \mathsf{m}_1^I(\{h\}) = 0.054,$$
$$\mathsf{m}_1^I(\{l,m\}) = 0.003, \mathsf{m}_1^I(\{l\}) = 0.167,$$
$$\mathsf{m}_1^I(\{m\}) = 0.189).$$

5 CONCLUSION

In this paper we have presented an event modelling and reasoning framework to represent and reason with uncertain event-observations from multiple

[7]Assume ordering (O, G, B).

sources such as low-level sensors. This approach provides rule-based systems to specify which event-observations to combine as well as to infer higher level inferred events from both primitive and combined event-observations. We demonstrate the applicability of our work using a real-world surveillance system scenario. In conclusion, we have found that it is important to correctly model, select and combine uncertain sensor information so that we obtain inferred events that are highly significant. This ensures appropriate actions can be taken to stop or prevent undesirable behaviours that may occur. As for future work, we plan to deal with partially matched information in the formula (condition) of inference rules. In other words, if a formula of a rule is met, this rule is triggered and an inferred event is generated. However, if a formula of multiple rules are partially met, then we need an approach to decide which rule should be triggered.

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